The Integral Test and Estimates of Sums

Solutions should show all of your work, not just a single final answer.

1. Use the integral test to determine whether or not the following series converge or diverge. In each case, first explain why the integral test can be applied.

   (a) $\displaystyle\sum_{n=1}^{\infty} \frac{n}{n^2+1}$

   (b) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n\sqrt{n}}$

   (c) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{\sqrt[3]{n}}$

2. Consider the series $\displaystyle\sum_{n=1}^{\infty} \frac{n}{3^n}$

   (a) Verify that the integral test can be used to decide if this series converges and then use the integral test to show it converges. (Hint: Use integration by parts to evaluate the integral.)

   (b) Determine an explicit upper bound for the remainder when estimating the series by the Nth partial sum, in terms of N.

   (c) Find an N for which the Nth partial sum has a remainder that is less than .001, and then compute that partial sum to 4 digits.

3. For each of the following infinite series, use the remainder bound from the integral test to find an N such that the Nth partial sum is within the specified distance from the series.

   (a) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^4}$; within .001.

   (b) $\displaystyle\sum_{n=3}^{\infty} \frac{1}{n(\ln n)^3}$; within .1.

4. T/F (with justification): If $f(x)$ is continuous, positive, and decreasing for $x \geq 1$, and $\displaystyle\int_1^{\infty} f(x)\,dx$ converges then $\displaystyle\sum_{n=1}^{\infty} f(n) = \int_1^{\infty} f(x)\,dx$.