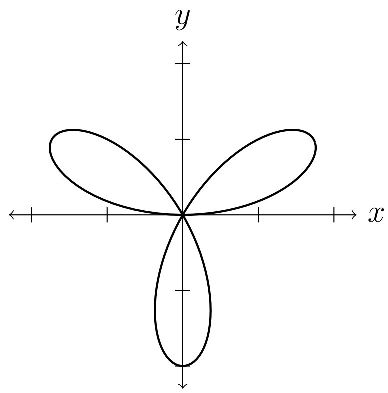
Areas in Polar Coordinates

Solutions should show all of your work, not just a single final answer.

1. Below is a 3-leaf rose $r = 2\sin(3\theta)$. Markings at ± 1 and ± 2 are on the x and y axes.



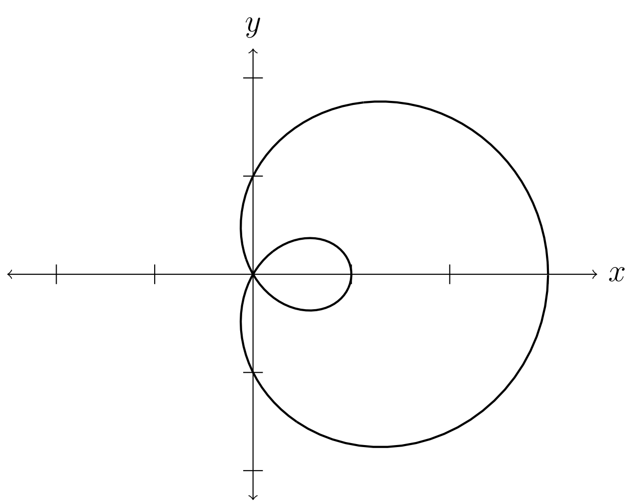
   (a) Show the point on the rose where $\theta = \pi/6$ is at the farthest distance from the origin by using the second derivative test.

   (b) Mark points on the rose where $\theta = 0$, $\pi/6$, $\pi/4$, $\pi/2$, and $3\pi/4$, and draw arrows on each leaf to indicate the direction in which it is traced out as θ increases.

   (c) Determine the smallest positive angle θ at which the rose passes through the origin, and use this to help you set up an integral for the area of the leaf in the first quadrant.

   (d) Compute the integral in part b. (Hint: Recall from Section 7.2 how to integrate $\sin^2 u$ by writing it in terms of $\cos(2u)$.)

2. Below is the graph of the limaçon $r = 1 + 2\cos\theta$. Unit markings are on the x and y axes.



(a) Mark the points on the curve where θ is $0, \pi/2, \pi, 3\pi/2$, and 2π, and use this to draw arrows on the curve (including on both the big and small loops) indicating the direction in which it is traced out as θ increases.

(b) Determine all θ between 0 and 2π where the graph passes through the origin.

(c) Set up, **but do not evaluate**, an integral in terms of θ for the area enclosed by the inner loop of the limaçon. Be sure the bounds of integration are right.

3. T/F (with justification)

If the polar curve $r = f(\theta)$ for $\alpha \leq \theta \leq \beta$ completely encloses a region, the area of the region is $\displaystyle\int_{\alpha}^{\beta} \frac{1}{2} f(\theta)^2 \, d\theta$.