University of Connecticut
Department of Mathematics

Math 1132 | Practice Exam 2 | Spring 2016

Name: ______________________________

Instructor Name: ______________________ Section: ________

TA Name: ______________________ Discussion Section: ________

Read This First!

- Please read each question carefully. Show **ALL** work clearly in the space provided. In order to receive full credit on a problem, solution methods must be complete, logical and understandable.
- Answers must be clearly labeled in the spaces provided after each question. Please cross out or fully erase any work that you do not want graded. The point value of each question is indicated after its statement. No books or other references are permitted.
- Calculators are allowed but you must show all your work in order to receive credit on the problem.

Grading - For Administrative Use Only

Page:	1	2	3	4	5	6	7	8	9	10	Total
Points:	12	9	8	9	11	12	12	12	8	7	100
Score:											

1. If the statement is always true, circle the printed capital T. If the statement is sometimes false, circle the printed capital F. In each case, write a careful and clear justification or a counterexample.

   (a) The geometric series $\sum_{n=4}^{\infty} \left(\frac{1}{3}\right)^n$ converges to $\frac{3}{2}$. (a) T F [2]

   (b) If $\lim_{n\to\infty} a_n = 0$ then the series $\sum_{n=1}^{\infty} a_n$ converges. (b) T F [2]

   (c) The series $\sum_{k=1}^{\infty} \frac{(-1)^k}{k^3}$ converges conditionally. (c) T F [2]

   (d) If $\sum_{n=1}^{\infty} |a_n|$ diverges then $\sum_{n=1}^{\infty} a_n$ diverges. (d) T F [2]

   (e) The fifth degree Taylor polynomial for $\sin(x)$ centered at $a = 0$ [2]

   is $x + \frac{x^3}{3!} + \frac{x^5}{5!}$. (e) T F

   (f) If the power series $\sum_{k=0}^{\infty} a_k\,(x-4)^k$ has a radius of convergence [2]

   equal to 2 then $\sum_{k=0}^{\infty} a_k$ diverges. (f) T F

2. For each multiple choice question, circle the correct answer. There is only one correct choice for each answer. No justification is required.

(a) Which of the following sequences is both bounded and monotonic? [3]

(i) $a_n = n^2$

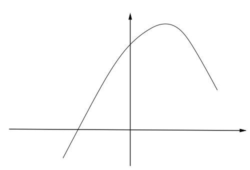
(ii) $a_n = \dfrac{n}{n+1}$

(iii) $a_n = \dfrac{\sin(\pi n)}{n}$

(iv) $a_n = \dfrac{n}{\sqrt{n+1}}$

(v) None of the above

(b) The function $f(x)$, whose graph is shown, has the Taylor polynomial of degree 2 centered at 0 given by $p_2(x) = a + bx + cx^2$. What can you say about a, b, c? (**Circle the correct answer for each part**) [3]



(i) a is: negative, zero or positive

(ii) b is: negative, zero or positive

(iii) c is: negative, zero or positive

(c) The value of the telescoping series $\displaystyle\sum_{k=1}^{\infty}\left(\frac{1}{k} - \frac{1}{k+1}\right)$ is [3]

(i) 0 (ii) 1 (iii) 2 (iv) 1/2 (v) None of the above

3. Consider the following series, all of which converge. For which of these series do you get a conclusive answer when using the **Ratio Test** to check for convergence? Write the letters of all possible answers. If no series satisfies this condition, write"none". You do not need to show your work. [8]

A $\displaystyle\sum_{k=1}^{\infty} \frac{k^6}{k!}$ **B** $\displaystyle\sum_{k=1}^{\infty} \frac{1}{(3k+4)^k}$ **C** $\displaystyle\sum_{k=2}^{\infty} \frac{\ln k}{k^2}$ **D** $\displaystyle\sum_{k=0}^{\infty} (-1)^k \frac{2}{5^k}$

4. Consider the following **sequences** and answer the questions that follow by circling all that apply.

(i) $a_n = \left(\dfrac{2n-1}{n+1}\right)^2$

(ii) $b_n = \dfrac{3^{n+5}}{2^n}$

(iii) $c_n = \sin(n)$

(iv) $S_n = \displaystyle\sum_{k=2}^{n} \frac{k}{k^3-2}$

(a) Which of the above sequences are bounded? (i) a_n (ii) b_n (iii) c_n (iv) S_n [3]

(b) Which of the above sequences are increasing? (i) a_n (ii) b_n (iii) c_n (iv) S_n [3]

(c) Which of the above sequences are convergent? (i) a_n (ii) b_n (iii) c_n (iv) S_n [3]

5. (a) Find the third degree Taylor polynomial $T_3(x)$ for $\cos x$ at $a = \frac{\pi}{4}$. [4]

(b) Use the 3rd degree Taylor polynomial from part a to approximate $\cos\left(\frac{3\pi}{16}\right)$. [4]

(c) Estimate the remainder for part b using the Taylor's inequality. [3]

(**Note:** If $|f^{n+1}(x)| \leq M$ for $|x-a| \leq d$, then the remainder $R_n(x)$ for the Taylor series satisfies the inequality $|R_n(x)| \leq \frac{M}{(n+1)!}|x-a|^{n+1}$ for $|x-a| \leq d$.)

6. (a) Determine the Taylor series of $f(x) = e^{-x^2}$ centered at 0. [4]

(b) Evaluate $\int e^{-x^2}\,dx$ as an infinite series. (**Remember to include +C**) [4]

(c) Use part (a) to determine the Taylor series of $f(x) = 2xe^{-x^2}$ centered at 0. [4]

7. Determine whether the following series converge conditionally, converge absolutely or diverge. Show your work in applying any tests used.

   (a) $\displaystyle\sum_{k=1}^{\infty} \frac{\sqrt{k^2+1}}{k}$ [4]

   (b) $\displaystyle\sum_{k=2}^{\infty} \frac{1}{k \ln k}$ [4]

   (c) $\displaystyle\sum_{k=0}^{\infty} \frac{4+3^k}{4^k}$ [4]

(d) $\displaystyle\sum_{k=1}^{\infty} \frac{k^4}{e^{3k}}$ [4]

(e) $\displaystyle\sum_{k=2}^{\infty} \frac{(-1)^k}{k^2-1}$ [4]

(f) $\displaystyle\sum_{k=1}^{\infty} \frac{(-1)^{k-1}}{\sqrt{k^2+1}}$ [4]

8. Find the interval of convergence of the power series $\displaystyle\sum_{n=1}^{\infty} \frac{1}{2^n n}(x-2)^n$. [6]

9. Consider the series $\displaystyle\sum_{n=1}^{\infty} \frac{3^n}{n} x^n$.

   (a) Find the radius of convergence, for this series. [2]

(b) Find the interval of convergence. [2]

(c) For which x does this series converge absolutely? [1]

10. How many terms of the series $\sum_{n=1}^{\infty}(-1)^{n-1}\frac{1}{(n+5)^2}$ do we need to add in order to find the sum of the series to within an accuracy of 0.001 (that is, $|\text{error}| < 0.001$)? [4]