1. If the statement is always true, circle the printed capital T. If the statement is sometimes false, circle the printed capital F. In each case, write a careful and clear justification or a counterexample.

   (a) If a force of $F(x) = 6x$ pounds is required to stretch a spring x feet beyond its rest length, then 36 ft-lbs of work is done in stretching the spring from its natural length to 6 feet beyond its rest length. T F

   (b) The trapezoid rule with $n = 5$ for $\int_0^4 \frac{dx}{2x+1}$ will be an overestimate. T F

   (c) $\ln(2.5) = \sum_{n=1}^{\infty} (-1)^{n-1} \frac{(1.5)^n}{n}$. T F

   (d) The improper integral $\int_1^{\infty} \frac{x^2}{(x^3+7)^{1/3}}\, dx$ converges. T F

   (e) The tangent line to the parametric curve $(x, y) = (t - 1/t, 4 + t^2)$ at the point where $t = 1$ has equation $y = x + 5$. T F

2. For each multiple choice question, circle the correct answer. There is only one correct choice for each answer.

   (a) A cylindrical tank with a radius of 1 meter and a height of 8 meters is half full. Letting $y = 0$ correspond to the top of the tank, the density of water be 1000 kg/m^3, and g be the acceleration due to gravity in m/sec^2, the work required to pump the water out of the tank is

   (i) $1000\pi g \int_4^8 y\, dy$ (ii) $1000\pi g \int_0^8 y\, dy$ (iii) $1000\pi g \int_0^4 y\, dy$ (iv) $16000\pi g \int_4^8 y\, dy$

   (b) The Taylor series at $x = 0$ for $\sin x$ is

   (i) $\sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{n!}$ (ii) $\sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!}$ (iii) $\sum_{n=1}^{\infty} (-1)^n \frac{x^{2n-1}}{(2n-1)!}$ (iv) $\sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!}$

   (c) A parametric curve tracing out the circle once clockwise for $0 \le t \le \pi$ starting at $(1, 0)$ is

   (i) $(\cos t, \sin t)$ (ii) $(\cos t, -\sin t)$ (iii) $(\cos(2t), \sin(2t))$ (iv) $(\cos(2t), -\sin(2t))$

3. This question is about the curve $y = \tan x$.

   (a) Write a definite integral that gives the arc length of curve $y = \tan x$ from $x = 0$ to $x = \pi/4$.

   (b) Write a definite integral that gives the area of the surface formed by revolving the curve $y = \tan x$ from $x = 0$ to $x = \pi/4$ around the x-axis.

4. Use the error bound formulas on the last page to determine an n such that the Trapezoid rule with n subintervals approximates $\int_0^1 \frac{1}{e^x}\, dx$ to within .001.

5. (a) Obtain the Taylor series for $\frac{1}{1+x}$ at $x = 0$ from the geometric series for $\frac{1}{1-x}$, writing your final answer in summation notation.

   (b) Use your result from part (a) and integration to write down the Taylor series at $x = 0$ for $\ln(1 + x)$ and then find the radius of convergence of that series.

6. (a) Find the 3rd-order Taylor polynomial centered at 4 for $\frac{1}{\sqrt{x}}$.

   (b) Use Taylor's inequality (see the last page) to give an upper bound for the error in approximating $\frac{1}{\sqrt{3.99}}$ by the polynomial in part (a) at $x = 3.99$.

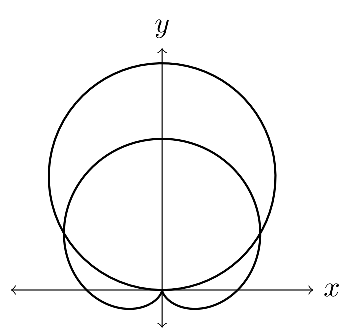
7. Use the error bound for alternating series to determine how many terms of the series $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^2+1}$ need to be added to estimate the full series with $|\text{error}| < 0.001$.

8. Use the Integral Test to show $\sum_{n=1}^{\infty} \frac{1}{n^p}$ converges if $p > 1$ and diverges if $0 < p < 1$.

9. Determine which of the following series converges conditionally, converges absolutely or diverges. Specify which convergence test you use and show how it leads to the answer.

   (a) $\sum_{n=2}^{\infty} \frac{(-1)^n}{\ln n}$ (b) $\sum_{n=1}^{\infty} \frac{n^2}{n^2+50}$ (c) $\sum_{n=1}^{\infty} \frac{(-1)^n n^5}{n!}$ (d) $\sum_{n=0}^{\infty} \frac{5}{2^n+5n+3}$

10. Below are graphs of $r = 3\sin\theta$ and $r = 1 + \sin\theta$.



   (a) Determine both polar and rectangular coordinates for all points where the curves cross in the first and second quadrants (not including the origin).

   (b) Set up, but do **not** evaluate, an integral for the area of the region inside $r = 3\sin\theta$ and outside $r = 1 + \sin\theta$.

11. Solve for y exactly: (a) $\frac{dy}{dx} = \frac{\sin x}{y^2}$ with $y(0) = 3$. (b) $\frac{dy}{dx} = y\cos x + xy$ with $y(0) = 3$.

12. Find the orthogonal trajectories of the family of curves $y = kx^4$, where k is an arbitrary (nonzero) constant.

13. A tank contains 30 L of water with 3 kg of salt dissolved in it. Brine that contains 5 kg of salt per liter enters the tank at a rate of 4 L/min. The solution in the tank is kept well mixed and is drained at a rate of 4 L/min. Use differential equations to determine how much salt remains in the tank after 30 minutes.

14. Compute $\int_0^{\infty} e^{-2x}\sin(x)\,dx$. Show all work.

15. Compute $\int \frac{dx}{x^2 - ax}$, where $a \neq 0$. Your answer will depend on a. Show all work.

Error Bound Formulas

Trapezoid Rule and Error Bound: Let $a = x_0 < x_1 < x_2 < \cdots < x_{n-1} < x_n = b$ with $x_i - x_{i-1} = \Delta x = \dfrac{b-a}{n}$ for all i. The nth approximation T_n to $\int_a^b f(x)\,dx$ using the Trapezoid rule is

$$T_n = (f(x_0) + 2f(x_1) + 2f(x_2) + \cdots + 2f(x_{n-2}) + 2f(x_{n-1}) + f(x_n))\frac{\Delta x}{2}$$

and the error in approximating the integral by T_n can be bounded as:

$$\left| \int_a^b f(x)\,dx - T_n \right| \leq \frac{K(b-a)}{12}(\Delta x)^2 = \frac{K(b-a)^3}{12n^2},$$

where K is any upper bound on $|f''(x)|$ over $[a,b]$: $|f''(x)| \leq K$ for $a \leq x \leq b$.

Taylor's Inequality: Let $T_n(x)$ be the nth-order Taylor polynomial for $f(x)$ at $x = a$. If $|f^{(n+1)}(x)| \leq M$ for $|x-a| \leq d$ then

$$|T_n(x) - f(x)| \leq M\frac{|x-a|^{n+1}}{(n+1)!} \quad \text{for} \quad |x-a| \leq d.$$