University of Connecticut
Department of Mathematics

MATH 1132 | PRACTICE EXAM 1 | SPRING 2016

NAME: ______________________________

Instructor Name: ______________________________ Section: __________

TA Name: ______________________________ Discussion Section: __________

Read This First!

- Please read each question carefully. Show **ALL** work clearly in the space provided. In order to receive full credit on a problem, solution methods must be complete, logical and understandable.
- Answers must be clearly labeled in the spaces provided after each question. Please cross out or fully erase any work that you do not want graded. The point value of each question is indicated after its statement. No books or other references are permitted.
- Calculators are allowed but you must show all your work to receive credit on the problem.
- Except for problems about approximate integration, give any numerical answers in exact form, **not as approximations**. For example, one-third is $\frac{1}{3}$, not .33 or .33333. And one-half of π is $\frac{1}{2}\pi$, not 1.57 or 1.57079.
- **Note**: This is only a sample exam. It illustrates the kinds of topics that will be on the midterm, but is no substitute for a thorough review (including your homework and worksheet problems). In particular, problems on the actual exam as well as the point distribution among those problems will not be identical to what you see in the sample exam.

Grading - For Administrative Use Only

Question:	1	2	3	4	5	6	7	8	Total
Points:	15	15	4	8	24	12	10	12	100
Score:									

1. If the statement is always true, circle the printed capital T. If the statement is sometimes false, circle the printed capital F. In each case, write a careful and clear justification or a counterexample.

   (a) If $\lim_{n\to\infty} a_n = L$, then $\lim_{n\to\infty} a_{2n+1} = L$. (a) T F [3]
   Justification:

   (b) Under Hooke's law, the work required to stretch a spring 2 inches beyond its natural length is twice that required to stretch it 1 inch beyond its natural length. (b) T F [3]
   Justification:

   (c) The trapezoid rule with $n = 10$ for $\int_0^4 x^3\,dx$ will be an underestimate of the integral's value. (c) T F [3]
   Justification:

   (d) If $\int_0^1 f(x)\,dx$ is an improper integral, then so is $\int_1^2 f(x)\,dx$. (d) T F [3]
   Justification:

   (e) The series $\sum_{n=1}^{\infty} \frac{1}{n}$ is divergent. (e) T F [3]
   Justification:

2. Determine whether each of the following improper integrals is convergent or divergent. For those that are convergent, give their exact value (not decimal approximations). Those determined to be divergent must have an explanation for their divergence.

   (a) $\displaystyle\int_1^\infty \frac{dx}{\sqrt{x}}$ [5]

   (b) $\displaystyle\int_0^\infty \frac{dx}{(2x+1)(x+4)}$ [5]

   (c) $\displaystyle\int_1^\infty \frac{\ln x}{x}\, dx$ [5]

3. Find a formula for the general term a_n of the sequence assuming that the pattern of the first few terms continues.

   (a) $\left\{\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \frac{1}{32}, \frac{1}{64}, \ldots\right\}$ [2]

   (b) $\left\{\frac{1}{2}, -\frac{4}{3}, \frac{9}{4}, -\frac{16}{5}, \frac{25}{6}, -\frac{36}{7}, \ldots\right\}$ [2]

4. Determine whether the sequence convergences or diverges. If it converges find the limit.

   (a) $a_n = \dfrac{n^4}{n^3 - 2n}$ [4]

   (b) $a_n = \dfrac{\cos^2 n}{2^n}$ [4]

5. Determine if the series is convergent or divergent. If it is convergent find its sum.

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{2+n}{1-2n}$ [6]

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{2^n+4^n}{e^n}$ [6]

(c) $\displaystyle\sum_{n=1}^{\infty} \left(\frac{3}{5^n}+\frac{1}{n^2}\right)$ [6]

(d) $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n^2-n}$ [6]

6. (a) A force of 30 N is required to maintain a spring stretched from its natural length of 12 cm to a length of 15 cm. How much work is done to stretch the spring from 12 cm to 20 cm? [4]

(b) A cylindrical keg of root beer with height 1 m and diameter $\frac{1}{4}$ m is *half-filled* and is standing upright (on one of its circular bases). The density of the root beer is 2000 kg/m^3. Set up, but do **NOT** evaluate, an integral that is equal to the work required to pump the root beer out of the top of the keg. [4]

(c) Consider the same half-full cylindrical keg of root beer with height 1 m and diameter $\frac{1}{4}$ m as in part b, but set the keg down on its side. If a hole is made along the (new) top of the rotated keg, set up but do **NOT** evaluate an integral equal to the work required to pump the root beer out. (Hint: Start by drawing a good picture.) [4]

7. Use the error bound formulas on the last page to determine an n such that the trapezoid rule with n subintervals approximates $\int_0^1 \frac{1}{3^x}\,dx$ to within .001. [10]

8. (a) Compute the approximations T_4 and M_8 for $\int_1^3 \frac{1}{e^x}\,dx$. [4]

(b) Estimate the error in each of the approximations from part a. [4]

(c) How large do we have to choose n so that the approximations T_n and M_n to the integral in part (a) are accurate to within 0.0001? [4]

Approximate integration formulas

In the formulas below, $a = x_0 < x_1 < x_2 < \cdots < x_{n-1} < x_n = b$ with $x_i - x_{i-1} = \Delta x = \dfrac{b-a}{n}$ for all i.

Midpoint Rule and Error Bound:

$$M_n = (f(\overline{x}_1) + f(\overline{x}_2) + f(\overline{x}_3) + \cdots + f(\overline{x}_n))\Delta x$$

and

$$\left| \int_a^b f(x)\,dx - M_n \right| \leq \frac{K(b-a)}{24}(\Delta x)^2 = \frac{K(b-a)^3}{24n^2},$$

where $\overline{x}_i$ is the midpoint of $[x_{i-1}, x_i]$ and K is an upper bound on $|f''(x)|$ over $[a,b]$: $|f''(x)| \leq K$ for $a \leq x \leq b$.

Trapezoid Rule and Error Bound:

$$T_n = (f(x_0) + 2f(x_1) + 2f(x_2) + \cdots + 2f(x_{n-2}) + 2f(x_{n-1}) + f(x_n))\frac{\Delta x}{2}$$

and

$$\left| \int_a^b f(x)\,dx - T_n \right| \leq \frac{K(b-a)}{12}(\Delta x)^2 = \frac{K(b-a)^3}{12n^2},$$

where K is an upper bound on $|f''(x)|$ over $[a,b]$: $|f''(x)| \leq K$ for $a \leq x \leq b$.

Simpson's Rule and Error Bound:

$$S_n = (f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + \cdots + 2f(x_{n-2}) + 4f(x_{n-1}) + f(x_n))\frac{\Delta x}{3}$$

and

$$\left| \int_a^b f(x)\,dx - S_n \right| \leq \frac{K(b-a)}{180}(\Delta x)^4 = \frac{K(b-a)^5}{180n^4},$$

where n is even and K is an upper bound on $|f^{(4)}(x)|$ over $[a,b]$: $|f^{(4)}(x)| \leq K$ for $a \leq x \leq b$.