
7.2 Trigonometric Integrals

Trigonometric Identities and Formulas.

1. $\sin^2 \theta + \cos^2 \theta = 1$ 3. $\sin 2\theta = 2 \sin \theta \cos \theta$ 5. $\cos^2 \theta = \frac{1 + \cos 2\theta}{2}$

2. $\tan^2 \theta + 1 = \sec^2 \theta$ 4. $\cos 2\theta = \cos^2 \theta - \sin^2 \theta$ 6. $\sin^2 \theta = \frac{1 - \cos 2\theta}{2}$

Example: Evaluate $\int_0^\pi \sin^3(5x) dx$.

Thinking about the problem:

To integrate a power like $\sin^3(5x)$, let's write $\sin^3 \theta$ in terms of lower powers. By the first trigonometric identity above, we can write $\sin^2 \theta = 1 - \cos^2 \theta$, so

$$\sin^3 \theta = \sin^2 \theta \sin \theta = (1 - \cos^2 \theta) \sin \theta.$$

Therefore (using $\theta = 5x$)

$$\int_0^\pi \sin^3(5x) dx = \int_0^\pi (1 - \cos^2(5x)) \sin(5x) dx.$$

Doing the problem:

After rewriting of the function being integrated, let's use the substitution $u = \cos(5x)$, so $du = -5 \sin(5x) dx$:

$$\int (1 - \cos^2(5x)) \sin(5x) dx = \int (1 - u^2) \frac{-du}{5} = -\frac{1}{5} \int (1 - u^2) du.$$

Let's turn x -bounds into u -bounds in the definite integral:

$$x = 0 \implies u = \cos(5 \cdot 0) = \cos 0 = 1, \quad x = \pi \implies u = \cos(5\pi) = -1.$$

Therefore

$$\begin{aligned}\int_0^\pi \sin^3(5x) dx &= \int_{x=0}^{x=\pi} (1 - \cos^2(5x)) \sin(5x) dx \\ &= -\frac{1}{5} \int_{u=1}^{u=-1} (1 - u^2) du \quad (\text{Note the order of integration}) \\ &= \frac{1}{5} \int_{-1}^1 (1 - u^2) du \quad (\text{Sign change in the order of integration}) \\ &= \frac{1}{5} \left(u - \frac{u^3}{3} \right) \Big|_{-1}^1 \\ &= \frac{1}{5} \left(\left(1 - \frac{1}{3} \right) - \left(-1 - \frac{-1}{3} \right) \right) \\ &= \frac{1}{5} \left(1 - \frac{1}{3} + 1 - \frac{1}{3} \right) \\ &= \frac{1}{5} \left(2 - \frac{2}{3} \right) \\ &= \frac{4}{15}.\end{aligned}$$

Solutions should show all of your work, not just a single final answer.

1. Identify the trigonometric identities to simplify the following integrands, and carry out the integration. Remember to include $+C$ in the final answer.

(a) $\int \sin^2 x \, dx$

(b) $\int \cos^3 x \, dx$

2. Evaluate the definite integral $\int_0^{\pi/2} \sin^4 x \cos^3 x \, dx$.

3. Evaluate $\int \sin^2 x \cos^2 x \, dx$ by using a trigonometric identity involving $\sin x \cos x$ to simplify the integrand.

4. Evaluate the integral $\int \sec^4 x \, dx$.

5. Evaluate $\int \tan^3 x \sec^3 x \, dx$

6. T/F (with justification): The value of $\int_{-\pi}^{\pi} \sin^9 x \, dx$ is 0.