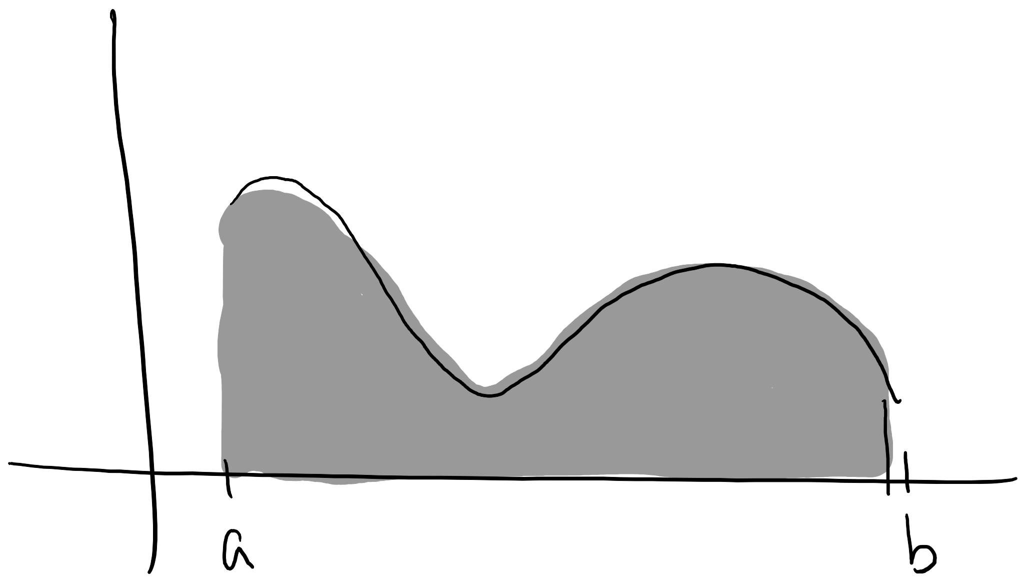
Section 5.1: Areas and Distances

(1) In this section, we focus on finding the area under curves. When we say "area under the curve" we really mean the area between the curve and the x-axis. Draw several curves and shade in the area under the curve in each case.



(2) How can we approximate the area under a curve using rectangles? (Note: a rectangular approximation is called a Riemann Sum) What decision do we need to make? How do we get a better and better approximation?

> We split the region into n rectangles of equal width. Once we know how many rectangles there are we know what each of their width should be ($\frac{b-a}{n}$). Then we need to find their height. To do this we use either left-hand, right-hand or mid-points of the interval and then we find the function value at that point.

(3) What does it mean to have an "over-estimate" or an "under-estimate"? How can you tell if a right hand sum gives an over estimate or an underestimate? Can you always tell for all functions?

> An approximation is an overestimate is it gives an number which is larger than the actual value. It is an underestimate if it gives a value which is lower than the actual value. A right hand-sum is an underestimate is the function is decreasing and an over-estimate if the function is increasing. If a function is both increasing and decreasing on an interval, we might not be able to tell if a right or left hand sum is an overestimate or an underestimate.

(4) If you know the (constant) velocity of a car, how can you find its distance? What if the velocity is changing? Can you still find or approximate the distance?

Given a car's velocity, we multiply by time to find the distance traveled. (e.g. 5mph times 3 hours = 15 miles). If the velocity is changing, we can still assume it is constant over small time intervals and then do the velocity times the distance.

(5) Compare the process of approximating the distance traveled of a car given its velocity and estimating the area under a curve using rectangles. What do you notice?

When we are approximating the distance traveled, we are assuming the velocity is constant for a short time (so its a straight vertical line) and then multiplying it by the width of the interval - this is exactly the same as finding the area of a rectangle under the curve. We get a better approximation was the width go to 0 (this assume the function is constant on smaller and smaller regions). Thus finding the distance traveled is exactly the same as finding the area under the curve.

Extra Practice in Book: 5.1: 2, 3, 5, 13, 15