Section 4.4: Indeterminate forms and l'Hospital's Rule

(1) In this section, we learn about various indeterminate forms and how we can use l'Hospital's rule to help evaluate limits of these forms. What does l'Hospital's rule say? When can we use it?

> If we have an indeterminate form of $0/0$ or ∞/∞ type, then l'Hospital's rules says that
> $$\lim x \to a \frac{f(x)}{g(x)} = \lim_{x \to a} \frac{f'(x)}{g'(x)}$$
> as long as the second limit exists. To use this, we must first check that we have $0/0$ or ∞/∞ and then we take the derivative of the top and bottom separately and then apply the limit to the derivatives.

(2) What is an indeterminate form? There are 7 examples given in this section. What are they?

> An indeterminate from is a form we get when trying to take the limit which, just from the form, we don't don't what the limit is equal to. The examples we saw are:
> - $0/0$
> - ∞/∞
> - $0 \cdot \infty$
> - $\infty - \infty$
> - 1^{∞}
> - 0^0
> - ∞^0

(3) The following forms are NOT indeterminate. For each form below, say what a limit having that form will evaluate to.

(a) What does it mean that these forms are NOT indeterminate?

> If we are trying to evaluate a limit and get one of these forms when we do a substitution, then we know what the function will evaluate to.

(b) $\frac{0}{\infty} \to 0$

(c) $\frac{\infty}{0} \to \pm\infty$

(d) $\infty * +\infty * \to \infty^*$

(e) $0^1 \to 0$

(f) $0^{\infty *} \to 0$

*Usually, when we write ∞ in these forms, it could be replaced with $+\infty$ or $-\infty$, these cases specifically refer to $+\infty$.

(4) We know we can use l'Hopital's rule directly when evaluating limits of the form $\frac{0}{0}$ or $\frac{\infty}{\infty}$. What can we do if we have an indeterminate product?

> If you have an indeterminate product from two functions $f \cdot g$, rewrite this as $\frac{f}{1/g}$ or $\frac{g}{1/f}$. This will be an indeterminate quotient so we can use l'Hopital's rule.

(5) What can we do if we have an indeterminate difference?

> If you have an indeterminate difference, use algebra (common denominator factoring, etc) to get it to be an indeterminate quotient (or product then use method above).

(6) What can we do if we have an indeterminate power? (Note: There are a few concepts/techniques that come up multiple times in the course. One of them is the following: "If you have an x or function of x in the exponent that you need to "deal with," take the ln of both sides. This will allow us bring down the function of x". We saw in this in section 3.6. In this section, we see this technique used for taking limits.)

> Set $y =$ to the limit. Take the ln of both sides. Since ln is a continuous function, we can move it inside the limit. This allows us to bring down the power. We end up with an indeterminate product, which we can deal with as above.

Extra Practice in Book: 4.4: 1, 2, 3, 4, 9, 11, 21, 23, 27, 43, 47, 53, 57, 61, 86