Section 4.3: How Derivatives Affect the Shape of a Graph

(1) In this section, we learn how to use the first and second derivative of a function to understand the shape of its graph. Fill out the table below as completely as possible.

f	f'	f''
positive	no information	no information
negative	no information	no information
increasing	positive	no information
critical point	0 or undefined	no information
decreasing	negative	no information
concave up	increasing	positive
might be point of inflection	critical point	0 or undefined
concave down	decreasing	negative

Note: in order to have an inflection point, the second derivative must be 0 or undefined but also must change sign.

(2) When looking for local minimum or maximum, what points do we need to check? How to do know if we have a local minimum or local maximum?

We need to check the x-values where the derivative is 0 or undefined (the critical values). If at these values the derivative switches from positive to negative, we get a local maximum. If it switches from negative to positive, we get a local minimum.

(3) What is the definition of an inflection point? How does this definition compare to the definition of a critical point?

An inflection point is a point on the graph of the function where the function changes from concave up to concave down (or vice versa). To be an inflection point, it is not ebough for the second derivative to be 0. The sign must change. To be a critical point, you just need the first derivative to be 0.

(4) How can we use the second derivative of a function to determine if a critical point is a local min or max? When does this test fail?

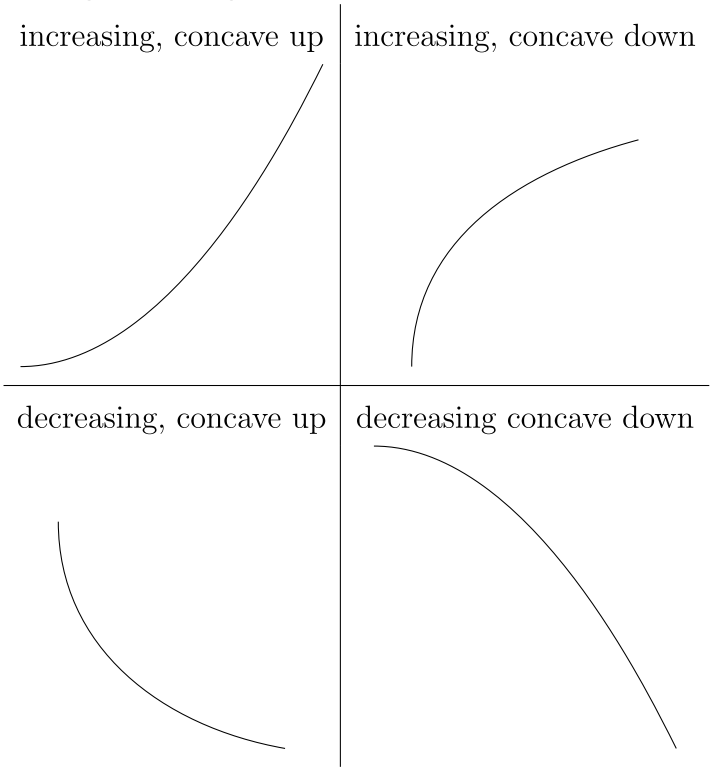
If a function is concave up at some critical point ($f''(x) > 0$) then a critical point must correspond to a local minimum. If it is concave down at the point, the critical point must be a local maximum. If the second derivative is 0, the point might be a local max, might be a local min or could be neither.

(5) When we are looking for increasing/decreasing intervals (or concave up/concave down intervals) we find the points where the derivative (second derivative) is 0 or undefined. When use these values to split our domain into intervals. Then what do we do? What numbers do we plug in where? Why does this work? Why do we know that on those intervals the function is either always increasing or always decreasing (concave up/concave down)?

When we are determining the decreasing/increasing intervals for a function, we want to know when the derivative is positive or negative. We know the derivative can only switch from positive to negative if it passes through 0 or is undefined so we find where the derivative is 0 or undefined. These split the domain into intervals of constant sign (of the derivative). To determine the sign of the derivative on each of those segments, we pick some point in the interval and plug it into the derivative to see if it is positive or negative. This tells us if the function is increasing or decreasing.

When we are determining the concavity intervals for a function, we want to know when the second derivative is positive or negative. We know the second derivative can only switch from positive to negative if it passes through 0 or is undefined so we find where the second derivative is 0 or undefined. These split the domain into intervals of constant sign (of the second derivative). To determine the sign of the second derivative on each of those segments, we pick some point in the interval and plug it into the second derivative to see if it is positive or negative. This tells us if the function is concave up or concave down.

(6) Draw segments of graphs of functions in each of the following cases:



Extra Practice in Book: 4.3: 3, 4, 5, 6, 7, 8, 11, 17, 19, 24, 29, 33, 35, 43, 57,