Section 3.2: The Product and Quotient Rules

(1) In this section, we learn the product and quotient rules. Write out the formula for each rule and then explain them in your own words.

Let f and g be differentiable at x. Then

$$(fg)'(x) = f'(x)g(x) + f(x)g'(x)$$

$$\left(\frac{f}{g}\right)'(x) = \frac{f'(x)g(x) - f(x)g'(x)}{g(x)^2} \qquad \text{(provided } g(x) \neq 0\text{)}.$$

Given a product (resp. quotient) of two differentiable functions, the product (resp. quotient) rule allows one to differentiate the product (resp. quotient). For the product rule, we take the derivative of the first function and multiply it by the second and then leave the first alone and only take the derivative of the second. And then add these to things. For the quotient rule, we do "low d-hi minus hi d-lo" all over the bottom squared.

(2) Do you need to product rule to compute the derivative of $f(x) = 3x^2$? Explain your reasoning.

No, since the 3 in a constant, we can just pull it out and then take the derivative of x^2.

(3) Do you need to quotient rule to compute the derivative of $f(x) = \dfrac{3x^2+1}{4}$? Explain your reasoning.

No. Dividing by 4 is the same as multiplying by 1/4 so we can view this as $\frac{1}{4}(3x^2+1)$ and then since the 1/4 is a multiple we can just pull it out and then take the derivative of the inside.

(4) Find the derivative of $f(x) = (x^2 - 3x)(x + 2)$ both with and without using the product rule.

By product rule:

$$\begin{aligned}\big((x^2-3x)(x+2)\big)' &= \big(x^2-3x\big)'(x+2) + (x^2-3x)(x+2)' \\ &= (2x-3)(x+2) + (x^2-3x) \\ &= 3x^2 - 2x - 6;\end{aligned}$$

by distributing first:

$$\big((x^2-3x)(x+2)\big)' = \big(x^3 - x^2 - 6x\big)' = 3x^2 - 2x - 6.$$

(5) Find the derivative of $f(x) = \dfrac{2x^2 + \sqrt{x} + 1}{x}$ both with and without using the quotient rule.

By quotient rule:

$$\begin{aligned} f'(x) &= \frac{(2x^2 + \sqrt{x} + 1)'x - (2x^2 + \sqrt{x} + 1)(x)'}{x^2} \\ &= \frac{(4x + \frac{1}{2}x^{-1/2})x - (2x^2 + \sqrt{x} + 1)}{x^2} \\ &= 2 - \frac{1}{2x^{3/2}} - \frac{1}{x^2}; \end{aligned}$$

by distributing first:

$$f'(x) = \left(2x + x^{-1/2} + x^{-1}\right)' = 2 - \frac{1}{2x^{3/2}} - \frac{1}{x^2}.$$

Extra Practice in Book: 3.2: Derivative Rules (3-30) until comfortable with all rules. 31, 35, 41, 42, 44, 45, 49, 51,