Section 3.10: Linear Approximations and Differentials

(1) In this section, we explore how we can use the tangent line to a function $f(x)$ at a point $x = a$ to approximate the function. Explain why this works. Can the tangent line be uses as a good approximation for the entire function?

We know that near $x = a$ the tangent line is close to the function. So we can use the tangent line to approximate the function, but only near $x = a$

(2) Are the linearization of a function and its tangent line different? Explain your reasoning.

We sometimes write the linearization as $f(x) =$ something and the tangent line as $y =$ something but they are equal to the same thing as long as they are both at $x = a$ for the same a value. The linearization just gives us a linear function which approximates the function near $x = a$ which is exactly what the tangent line does.

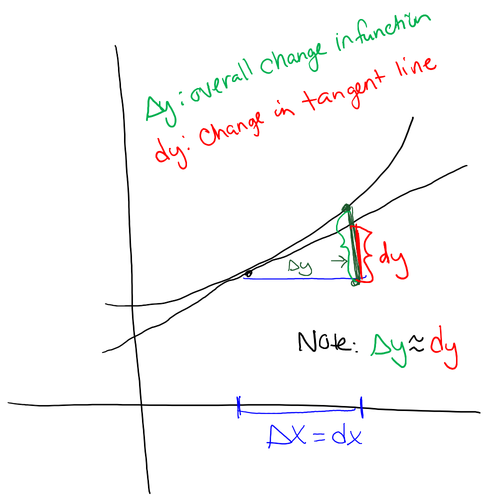
(3) If we are using the tangent line to a function $f(x) = \sqrt{x+3}$ to approximate $\sqrt{4.2}$, at what x value should we find the tangent line and what value should we plug in for x when we are done?

If we want to approximate $\sqrt{4.2}$ then we want $\sqrt{x+3} = \sqrt{4.2}$ so $x+3 = 4.2$ which means $x = 1.2$. Thus we should probably use a tangent line approximation at $x = 1$ since its a nice number near $x = 1.2$ and when we are done plug $x = 1.2$ into the linearization.

(4) We also talk about differentials in this section. Explain the connection between Δx, dx, Δy and dy. Illustrate with a sketch. How do we find dy?

The values Δx represents a small change in x. The corresponding quantity Δy represents how much the function values (y value) changes when x changes by Δx. The value dx also represents a small change in x but it corresponding quantity dy represents how much the tangent line changes instead of how much the function changes. Since the tangent line is an approximation of the function, dy can be used to approximate Δy. We find dy by doing

$$dy = f'(x)dx.$$



(5) Explain how we can use differentials to estimate error bounds. Define the terms relative error and percentage errors. Why are they important?

When we are finding error bounds, we want to know how much the function y might change with some error in the input (x). So we want to find Δy. As above, we can use dy to estimate Δy.

If we have an error of 100 thats a much bigger deal if our numbers are like 150 than if they are like 10000. So the relative error is just a measure of the error compared to the actual value. To find this we do $\frac{\Delta y}{y}$ which can be approximated by $\frac{dy}{y}$. If we express this same quantity as a percent, we get percent error.

For the numbers we had above, this would look like:

$$\Delta y = 100, y = 150$$

Then relative error is $\frac{100}{150} = 2/3 \approx .6667$ and percent error is 66.67%.

$$\Delta y = 100, y = 10000$$

Then relative error is $\frac{100}{10000} = .01$ and percent error is 1%.

Extra Practice in Book: 3.10: 3, 5, 13, 21, 25, 27, 33