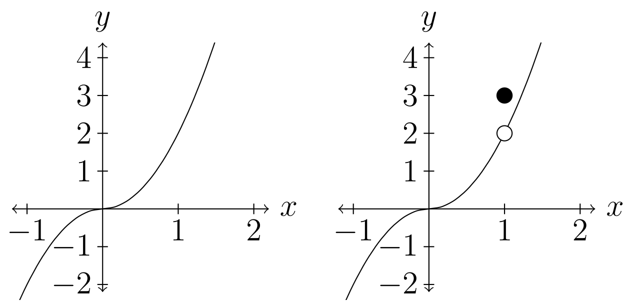
Section 2.2: The Limit of a Function

(1) What is the intuitive definition of a limit? Explain it in your own words.

> The limit of a functions tells the what the y values of the function are approaching as x gets closer and closer to a (but not equal to a)

(2) For the function graphed on the left, what is $\lim_{x \to 1} f(x)$? What would happen if we changed $f(1) = 3$ but kept everything else the same? Does the limit change? (See graph on right).



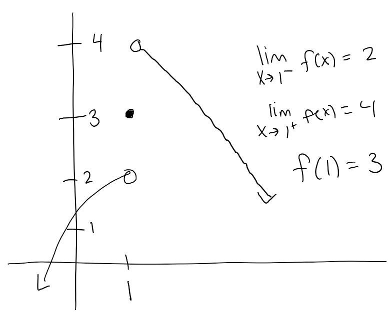
> The limit of both functions is 2. The limit does not care about what the function value is.

(3) Explain how you can use a calculator to approximate a limit. Why do you need to be careful?

> To find the limit as x approaches a, you can put in number closer and closer to a and see what the outputs are approaching. But sometimes, the calculator can mislead you due to rounding error. (See examples in the book.) Also try $\lim_{x\to\infty}(1+\frac{5}{x})^x$

(4) Sometimes, different things can happen to a function on either side of an x-value. We can explain this behavior in (mathematical) words using one sided limits. What does $\lim\limits_{x \to a^+}$ and $\lim\limits_{x \to a^+}$ mean? Practice drawing functions that satisfy different one sided limit values and function values.

$lim_{x \to a^+} f(x)$ gives the right handed limit. It tells what happens to the function as x approaches a from the right (numbers bigger than a) $lim_{x \to a^-} f(x)$ gives the left hand limit. Draw several examples of piecewise functions.



(5) A limit can only exist at a point if the two one sided limits exist at that point. Another way to write this is:

$\lim_{x \to a} f(x)$ exists if and only if $lim_{x \to a^+} f(x) = lim_{x \to a^-} f(x)$

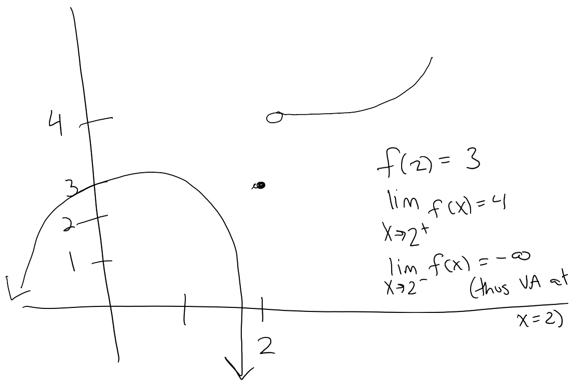
(6) What does it mean for a limit to equal ∞ or $-\infty$?

For the limit to equal ∞, the function must get larger and larger (larger than any finitie number) has x apporaches a. For $-\infty$, the function values mustg get smaller and smaller so that its smaller than any finite negative number as x appraches a.

(7) There are 6 different things that could happen that give us a vertical asymptote. What are they?

The right hand limit could be ∞ or $-\infty$ (without the left hand limit matching it). Or the left hand limit could be ∞ or $-\infty$ (without the right hand limit matching it). Or the limit could be ∞ or $-\infty$ (which means the left and right hand limits match.

(8) Draw the graph of a function with $f(2) = 3$, $\lim_{x \to 2^+} f(x) = 4$ and that has a vertical asymptote at $x = 2$.



Important ideas to know from this section:

- Limits don't care about the value of the function at the specific x value. They care about what happens one either side of that x-value.

Extra Practice in Book: 2.2: 5, 9, 11, 17, 21, 33