University of Connecticut
Department of Mathematics

MATH 1131 PRACTICE EXAM 2 SPRING 2019

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SIGNATURE: ____________________

Instructor Name: ____________________ Lecture Section: ________

TA Name: ____________________ Discussion Section: ________

Sections Covered: 3.2, 3.3, 3.4, 3.5, 3.6, 3.8, 3.9, 3.10, 4.8

Read This First!

- Please read each question carefully. All questions are multiple choice. There is only one correct choice for each answer. Each question is one point.

- Indicate your answers on the answer sheet. The answer sheet is the **ONLY** place that counts as your official answers.

  (1) When you're done, hand in **both** the exam booklet and the answer sheet.

  (2) You will receive the exam booklet back after the exam is graded. The booklet is not graded, but you may circle answers there for your records.

- Calculators are allowed **below the level of TI-89**. In particular, **TI-Nspire is not allowed**. No books or other references are permitted.

1. Determine $f'(1)$ for the function $f(x) = (x^3 - x^2 + 1)(x^4 - x + 2)$.

(A) 3 (B) 0 (C) 4

(D) 2 (E) 5

$F'(x) = (x^3 - x^2 + 1)(4x^3 - 1) + (3x^2 - 2x)(x^4 - x + 2)$

$f'(1) = (1-1+1)(4-1) + (3-2)(1-1+2)$

$= 5$

2. Find the equation of the tangent line to the curve $y = \dfrac{x}{x+1}$ at $x = 1$.

(A) $y = \frac{1}{2}$ (B) $y = -\frac{1}{2}x + 1$ (C) $y = \frac{1}{2}x$

(D) $y = -\frac{1}{4}x + \frac{3}{4}$ (E) $y = \frac{1}{4}x + \frac{1}{4}$

$y' = \dfrac{(x+1)\cdot 1 - (x)(1)}{(x+1)^2}$

$y @ x=1 \Rightarrow \dfrac{1}{1+1} = \dfrac{1}{2}$

$y'\big|_{x=1} = \dfrac{2-1}{(2)^2} = \dfrac{1}{4}$

point $(1, 1/2)$ $m = 1/4$

$y = \frac{1}{4}(x-1) + \frac{1}{2} = \frac{1}{4}x + \frac{1}{4}$

3. If $f(x) = \sin(x)$, determine $f^{(125)}(\pi)$.

(A) 1 (B) -1 (C) 0

(D) 1/2 (E) $\sqrt{2}/2$

$f'(x) = \cos x$

$f^{2}(x) = -\sin x$

$f^{3}(x) = -\cos x$

$f^{4}(x) = \sin x$

Need to recognize the pattern on how the derivatives are being repeated

so $125 = 4 \cdot 31 + 1$

$f^{(125)}(x) = \cos x$

$f^{(125)}(\pi) = \cos(\pi) = -1$

4. To compute the derivative of $\sin^2 x$ with the chain rule by writing this function as a composition $f(g(x))$, what is the "inner" function $f(x)$? [1]

(A) x (B) x^2 $\boxed{\text{(C) } \sin x}$

(D) $\sin^2 x$ (E) None of the above

inside

outside $f(g(x))$

$g(x) = \sin x$ $f(x) = x^2$

$f(g(x)) = (\sin x)^2 = \sin^2 x.$

Side Note:

$\frac{d}{dx}(\sin x)^2 = 2\sin x \cdot \frac{d}{dx}\sin x$

$= 2\sin x \cdot \cos x.$

5. Let $y = f(x)g(x)$. Using the table of values below, determine the value of $\frac{dy}{dx}$ when $x = 2$. [1]

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	5	2	4	4
2	3	4	1	3
3	2	3	2	2
4	4	1	5	5
5	1	5	3	1

(A) 9 (B) 12 $\boxed{\text{(C) 13}}$

(D) 15 (E) 23

$y' = f'(x) \cdot g(x) + f(x) \cdot g'(x)$

$y'(x=2) = f'(2) \cdot g(2) + f(2) \cdot g'(2)$

$= 4 \cdot 1 + 3 \cdot 3 = 13$

6. Determine $f'''(\pi/2)$ for the function $f(x) = 2\sin x - 3\cos x$. [1]

(A) 2 (B) -2 (C) 3

$\boxed{\text{(D) } -3}$ (E) 1

$f(x) = 2\sin x - 3\cos x$

$f'(x) = 2\cos x - 3(-\sin x)$

$f'(x) = 2\cos x + 3\sin x$

$f^2(x) = -2\sin x + 3\cos x$

$f^3(x) = -2\cos x - 3\sin x$

$f^3(\pi/2) = -2\cos(\pi/2) - 3 \cdot \sin(\pi/2)$

$= -3\sin(\pi/2) = -3 \cdot 1$

$= -3$

7. If $g(x) = \dfrac{ax+b}{cx+d}$, then $g'(1)$ is which of the following? Note: The numbers a, b, c, and d are constants. [1]

(A) $\dfrac{a+b-c-d}{c+d}$ $\boxed{\text{(B)}}$ $\dfrac{ad-bc}{(c+d)^2}$ (C) $\dfrac{a+b-c-d}{(c+d)^2}$

(D) $\dfrac{ad+bc}{c+d}$ (E) $\dfrac{ad+bc}{(c+d)^2}$

Focus on basic skill.

$$g'(x) = \frac{(cx+d)\cdot(a) - (ax+b)\cdot c}{(cx+d)^2}$$

$$g'(1) = \frac{(c+d)a - (a+b)c}{(c+d)^2}$$

$$\frac{\cancel{ca} + ad - \cancel{ac} - bc}{(c+d)^2} = \frac{ad-bc}{(c+d)^2}$$

8. For the function $f(x) = x^3 \arctan(x)$, which of the following is $f'(1)$?

(A) $\dfrac{3\pi}{4}$ $\boxed{\text{(B) } \dfrac{3\pi}{4} + \dfrac{1}{2}}$ (C) $\dfrac{1}{2}$

(D) $\dfrac{\pi}{4}$ (E) $3\tan(1) + \sec^2(1)$

$$f'(x) = 3x^2 \cdot \arctan x + x^3 \cdot \frac{1}{1+x^2}$$

$$f'(1) = 3(1)^2 \cdot \tan^{-1}(1) + \frac{1^3}{1+1^2}$$

$$= 3 \cdot \frac{\pi}{4} + \frac{1}{2}$$

9. On the curve $x^y = y^x$ with x and y both positive, $\frac{dy}{dx}$ is which of the following? [1]

(A) $\frac{1-\ln x}{1-\ln y}$ (B) $x^{y-x}\ln x$ (C) $(1-\ln x)\frac{y}{x}$

(D) $(1-\ln y)\frac{y}{x}$ $\boxed{\text{(E)}}$ $\left(\frac{y}{x}\right)\left(\frac{x\ln y - y}{y\ln x - x}\right)$

$x^y = y^x$

Take lns on both sides.

$\ln x^y = \ln y^x$

$y\ln x = x\cdot\ln y$.

Now take derivative w.r.t "x"

Do not ~~forget~~ forget product rule

$\left(y'\ln x + y\cdot\frac{1}{x}\right) = 1\cdot\ln y + x\cdot\frac{1}{y}y'$

Now it is ALGEBRA

$y'\ln x - \frac{x}{y}y' = \ln y - \frac{y}{x}$

$y'\left(\ln x - \frac{x}{y}\right) = \ln y - \frac{y}{x}$

$y' = \frac{\ln y - y/x}{\ln x - x/y} = \boxed{\frac{x\ln y - y}{y\ln x - x}\cdot\frac{y}{x}}$

10. Find $\frac{d}{dx}\left[x^{\ln x}\right]$. [1]

(A) $(\ln x)x^{\ln x}$ (B) $2(\ln x)x^{(\ln x)+1}$ (C) $x^{(\ln x)-1}$

(D) $2(\ln x)x^{\ln x}$ $\boxed{\text{(E) } 2(\ln x)x^{(\ln x)-1}}$

$y = x^{\ln x}$

$\ln y = \ln x^{\ln x} = \ln x\cdot\ln x$

$= (\ln x)^2$

$\frac{1}{y}\cdot y' = 2\ln x\cdot\frac{1}{x}$

$y' = 2\cdot\ln x\cdot\frac{y}{x}$

$y' = 2\ln x\cdot\frac{x^{\ln x}}{x}$

$= 2\ln x\cdot x^{(\ln x - 1)}$

11. On the curve $xy^3 = x - y$, which of the following is $\frac{dy}{dx}$?

(A) $\frac{1-y^2}{1+2xy^2}$ (B) $\frac{1-y^3}{1-3xy^2}$ (C) $\frac{1+y^3}{1+3xy^2}$

(D) $\frac{1+y^2}{1+3xy^2}$ $\boxed{\text{(E) } \frac{1-y^3}{1+3xy^2}}$

$1 \cdot y^3 + x \cdot 3y^2 \cdot y' = 1 - y'$

$3xy^2 \cdot y' + y' = 1 - y^3$

$y'(3xy^2+1) = 1 - y^3$

$\longrightarrow y' = \frac{1-y^3}{(3xy^2+1)}$

12. The size of a colony of bacteria at time t hours is given by $P(t) = 100e^{kt}$, where P is measured in millions. If $P(5) > P(0)$, then determine which of the following is true. [1]

I. $k > 0$

II. $P'(5) < 0$

III. $P'(10) = 100ke^{10k}$

$\boxed{\text{(A) I and III only.}}$ (B) I and II only. (C) I only.

(D) II only. (E) I, II, and III.

$P(t) = 100\,e^{kt}$ is an exponential model, that is a soln to differential Eqn $\frac{dP}{dt} = k \cdot P$ with $P(0) = 100$

If $P(5) > P(0)$, then the population is growing so $k > 0$ $\boxed{\text{I-True}}$

$P'(t) = 100 \cdot k \cdot e^{kt}$; $P'(5) = 100k \cdot e^{5k} > 0 \Rightarrow \boxed{\text{II False}}$

$P'(10) = 100 \cdot k\, e^{10 \cdot k} \Rightarrow \boxed{\text{III True}}$

13. Suppose that the half-life of a certain substance is 20 days and there are initially 10 grams of the substance. The amount of the substance remaining after time t is given by [1]

(A) $10e^{10k}$ (B) $\ln(10)e^{kt/10}$ (C) $\ln(10)e^{t/10}$

$\boxed{\text{(D) } 10e^{-t\ln(2)/20}}$ (E) $10e^{t\ln(2)/20}$

Generally drawing a time line helps in these kinds of problems.

Let $A(t)$ be the amount of a substance

$A(0)=10$ $A(20)=5$

$t=0$ $t=20$ days

$A(t)=A_0e^{kt}$

$A(t)=10e^{kt}$

$A(t=20)=5$

$5=10e^{k\cdot 20}$

solve for k

$\frac{1}{2}=e^{20k}$

$k=\frac{1}{20}\ln\frac{1}{2}$

$k=-\frac{\ln 2}{20}$

$A(t)=10e^{-\frac{\ln 2}{20}\cdot t}$

14. Atmospheric pressure (the pressure of air around you) decreases as your height above sea level increases. It decreases exponentially by 12% for every 1000 meters. The pressure at sea level is 1013 hecto pascals. The amount of pressure at any height h is given by, [1]

(A) $1000e^{10h}$ (B) $\ln(1013)e^{kh/12}$ (C) $1013e^{\ln(0.88)/1000}$

(D) $1000e^{-h\ln(2)/20}$ $\boxed{\text{(E) } 101.3e^{h\ln(0.88)/1000}}$

$p(h)=101.3$

height
$h=0$
sea level.

$h=1000$

$p(h)=101.3e^{\frac{\ln(0.88)}{1000}\cdot h}$

$p(h)=p_0e^{kh}$

$p(1000)=89.144=101.3e^{k1000}$

$\Rightarrow k=\frac{\ln(0.88)}{1000}$

15. A particle moves along the curve $y = \sqrt[3]{x^4 + 11}$. As it reaches the point $(2, 3)$, the y-coordinate is increasing at a rate of 32 cm/s. Which of the following represents the rate of increase of the x-coordinate at that instant?

$\boxed{\text{(A) } 27 \text{ cm/s}}$ (B) 9 cm/s (C) 13.5 cm/s

(D) 6.75 cm/s (E) None of the above

$y = \sqrt[3]{x^4+11}$

$y^3 = x^4 + 11$

$3y^2 \cdot \frac{dy}{dt} = 4x^3 \cdot \frac{dx}{dt}$

$\frac{dx}{dt} = \frac{3y^2}{4x^3} \cdot \frac{dy}{dt}$

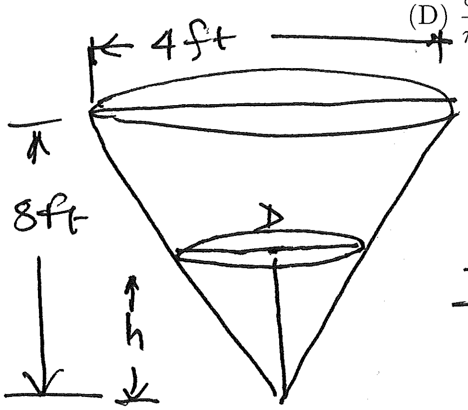
$x = 2,\ y = 3 \quad \frac{dy}{dt} = 32$

$\frac{dx}{dt} = \frac{3}{\cancel{4}} \cdot \frac{9}{\cancel{8}_{1}} \cdot \cancel{32}^{4} = 27$ cm/s

16. Water is withdrawn at a constant rate of 2 ft^3/min from an inverted cone-shaped tank (meaning the vertex is at the bottom). The diameter of the top of the tank is 4 ft, and the height of the tank is 8 ft. How fast is the water level falling when the depth of the water in the tank is 2 ft? (Remember that the volume of a cone of height h and radius r is $V = \frac{\pi}{3} r^2 h$?)

(A) $\frac{2}{\pi}$ ft/min (B) $\frac{4}{\pi}$ ft/min (C) $\frac{6}{\pi}$ ft/min

(D) $\frac{8}{\pi}$ ft/min (E) $\frac{16}{\pi}$ ft/min



$\frac{dh}{dt} = ?$ when $h = 2$ft

$V = \frac{\pi}{3} r^2 h. \quad r = \frac{D}{2}$

$\frac{D}{h} = \frac{4}{8} \quad D = \frac{1}{2} h.$

$V = \frac{\pi}{3} \cdot \left(\frac{D}{2}\right)^2 h$

$V = \frac{\pi}{3} \cdot \frac{D^2}{4} \cdot h \quad D = \frac{h}{2}$

$V = \frac{\pi}{3} \cdot \frac{1}{4} \cdot \left(\frac{h}{2}\right)^2 \cdot h$

$V = \frac{\pi}{3} \frac{h^3}{16} = \frac{\pi}{48} h^3$

$\frac{dV}{dt} = \frac{\pi}{48} \cdot 3h^2 \cdot \frac{dh}{dt}$

$2 = \frac{\pi}{\cancel{48}\ \cancel{16}\ 4} \cdot \cancel{3}^{1} \cdot \cancel{(2)^2}^{4} \cdot \frac{dh}{dt}$

$\boxed{\frac{8}{\pi} = \frac{dh}{dt}}$

17. Use the linearization for the function $f(x) = \sqrt{x^3 + 2x + 1}$ at $x = 1$ to approximate the value of $f(1.1)$.

(A) 2.0125 (B) 2.10 $\boxed{\text{(C) 2.125}}$

(D) 0.5 (E) 1.925

$L(x) = f(a) + f'(a)\cdot(x-a)$

$a = 1$ $f(x) = \sqrt{x^3+2x+1}$

$f'(x) = \dfrac{3x^2+2}{2\cdot\sqrt{x^3+2x+1}}$

$L(x) = 2 + \frac{5}{4}(x-1)$

$f(1) = 2$; $f'(1) = \frac{5}{4}$.

$L(1.1) = 2 + \frac{5}{4}(1.1-1)$

$= 2.125$

18. Let $f(x) = x^2 - 10$. If $x_1 = 3$ in Newton's method to solve $f(x) = 0$, determine x_2.

(A) 1/2 $\boxed{\text{(B) 19/6}}$ (C) 15/4

(D) 12/7 (E) 17/6

$f(x) = x^2 - 10$

$f'(x) = 2x$

$x_1 = 3$

$f(x_1) = f(3) = 3^2 - 10 = 1$

$f'(x_1) = f'(3) = 2\cdot 3 = 6$

$x_{n+1} = x_n \dfrac{f(x_n)}{f'(x_n)}$

$x_2 = x_1 - \dfrac{f(x_1)}{f'(x_1)}$

$= 3 - \left(\dfrac{-1}{6}\right)$

$x_2 = 19/6$