Math 1131 Week 2 Worksheet

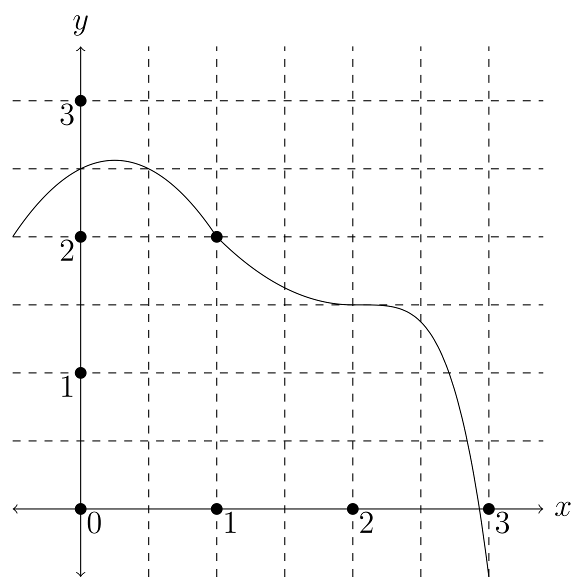
Name: ________________________

Discussion Section:_____________

Solutions should show all of your work, not just a single final answer.

2.4: The Precise Definition of a Limit

1. For the continuous function $f(x)$ whose graph is below, $f(1) = 2$. Estimate a value of $\delta > 0$ such that if $0 < |x - 1| < \delta$, then $|f(x) - 2| < 1/2$. Explain your answer by referencing the graph.



2. Let $g(x) = 4x - 1$. We will work towards showing that $\lim_{x \to 2} g(x) = 7$ by using the ε, δ definition of a limit.

(a) For $\varepsilon = 0.1$, find δ so that we get $|g(x) - 7| < \varepsilon$ whenever $|x - 2| < \delta$.

(b) For $\varepsilon = 0.01$, find δ so that we get $|g(x) - 7| < \varepsilon$ whenever $|x - 2| < \delta$.

(c) For $\varepsilon > 0$, find δ (in terms of ε) so that we get $|g(x) - 7| < \varepsilon$ whenever $|x - 2| < \delta$.

2.5: Continuity

3. Let

$$f(x) = \begin{cases} x^2 + x & \text{if } x < 1, \\ a & \text{if } x = 1, \\ x - 1 & \text{if } x > 1. \end{cases}$$

   (a) Determine the value of a for which $f(x)$ is continuous from the left at 1.

   (b) Determine the value of a for which $f(x)$ is continuous from the right at 1.

   (c) Is there a value of a for which $f(x)$ is continuous at 1? Explain.

4. Use the intermediate value theorem to show that there is a solution to $x - \sqrt{x} - \ln x = 0$ on the interval $(2, 3)$. Clearly explain your reasoning.

5. Let

$$f(x) = \begin{cases} 2 - kx & \text{if } x < 1, \\ k + x & \text{if } x > 1 \end{cases}$$

with the value of $f(1)$ to be determined.

(a) Compute $\lim_{x \to 1^-} f(x)$ in terms of k.

(b) Compute $\lim_{x \to 1^+} f(x)$ in terms of k.

(c) Find the values of k and $f(1)$ that make $f(x)$ continuous at $x = 1$.

(d) Using the choice of k and $f(1)$ in part (c), make a graph of $y = f(x)$ for $0 \leq x \leq 2$.

6. The function $f(x)$ is continous on the interval $(-3, 4)$. If we know that $f(-1) = 4$ and $f(3) = 7$, what can we say about the outputs of $f(x)$, i.e. what values does f definitely take and/or not take?

7. T/F (with justification) The function

$$f(x) = \begin{cases} \sin x & \text{if } x \leq 0, \\ 1 + \cos x & \text{if } x > 0 \end{cases}$$

has a jump discontinuity at $x = 0$.

8. T/F (with justification) A function that is continuous at a point has to be defined at the point.

9. T/F (with justification) A function that is discontinuous at a point can't be defined at the point.

2.6: Limits at Infinity and Horizontal Asymptotes

10. Find the limit in each case or explain why it does not exist (and if it is $\pm\infty$).

(a) $\lim_{x \to \infty} \dfrac{2x+3}{6x-7}$

(b) $\lim_{x \to -\infty} \dfrac{x^3}{\sqrt{6x^4-1}}$

(c) $\lim_{x \to \infty} \sqrt{x^2+3x} - x$

(d) $\lim_{x \to \infty} \dfrac{100000x}{x^3+x}$

(e) $\lim_{x\to\infty} \frac{\sqrt{16x^4 + 7x}}{8x^2 + 5}$

(f) $\lim_{x\to-\infty} \frac{e^{3x} - e^{-3x}}{e^{3x} + e^{-3x}}$

(g) $\lim_{x\to\infty} \sqrt{x} + \sin x$

(h) $\lim_{x\to\infty} \frac{1}{x} + \sin x$

11. Let $f(x) = \dfrac{\sqrt{4x^6+5}}{x^3-1}$.

   (a) Compute $\lim\limits_{x\to\infty} f(x)$.

   (b) Compute $\lim\limits_{x\to-\infty} f(x)$.

   (c) What are the horizontal asymptotes of the graph of $y = f(x)$?

   (d) What is the vertical asymptote of the graph of $y = f(x)$?

12. T/F (with justification) The graph of the function $y(x) = 3 + 6e^{-kx}$, with k a positive constant, has a horizontal asymptote $y = 6$.

13. T/F (with justification) If the continuous function $f(x)$ has domain $(-\infty, +\infty)$, then either $\lim\limits_{x\to\infty} f(x)$ exists or $\lim\limits_{x\to\infty} f(x)$ is $\pm\infty$.