Section 5.4: Indefinite Integrals and the Net Change Theorem

(1) In this section, we formally introduce the indefinite integral symbol to mean the antiderivative. Explain why it makes some level of sense to use the same symbol to means both "take the anti-derivative" and "find the area under the curve". How do we know which one we should do? What do we get when we do an indefinite integral (number, function, etc.)? A definite integral?

> If F is the anti-derivative of f, it means that the derivaitve or slope of F, at every point x is equal to the value of f. Now imagine we multiply that the slope, f, by a small $x - step$, dx. That product represents two different quantities. First it is the area of the rectangle with height approximately $f(x)$ and base spanning from x to $x + dx$. But it also represents how much F changes as we go from x to $x + dx$ since the slope of F times an x-step gives a y-step. Now, if we add up all the $f(x) * dx$ pieces as x increases from a to b, the first is the sum of the areas of the rectangles which, in the limit as dx goes to 0, is the area under the curve from a to b. This is the basic idea of the fundamental theorem of calculus.
> Now, using the fundamental theorem of calculus we know that to find the area under the curve, we need to find an antiderivative so we can view the integral symbol as a symbol to remind us to take the antiderivative. Thus we adopt this symbol as the "take the antiderivative" symbol. We can tell the difference between a definite and indefinite integral because the definite integral has bounds on the integral symbol.
> The definite integral is an number, however, indefinite integral is a family of functions which contains all the F which has derivative of f.

(2) Review the table of indefinite integrals in your book. Write out any ones you are not already comfortable with. Write down at least three examples in the form of the example provided.

e.g. The general antiderivative of $2x$ is the family of functions $x^2 + C$ since the derivative of $x^2 + C$ is 2x.

Function	Most General Antiderivative	Explain in words
$cf(x)$	$cF + C$	c times the antiderivative F, + C.
$f(x) + g(x)$	$F + G + C$	antiderivative of sum is sum of antiderivatives
$x^n (n \neq -1)$	$\frac{1}{n+1} x^{n+1} + C$	antiderivative of x^n is x^{n+1} over $(n+1)$ + C
$\frac{1}{x}$	$\ln x + C$	antiderivative of $\frac{1}{x}$ is $\ln(x)$ plus C
e^x	$e^x + C$	antiderivative of e^x is e^x plus C
b^x	$\frac{b^x}{\ln b} + C$	antiderivative of b^x is b^x divided by $\ln(b)$ plus C

Function: 5^x, General Antiderivative: $\frac{5^x}{ln5} + C$
Function: $\frac{1}{5}x^4$, General Antiderivative: $\frac{1}{25}x^5 + C$
Function: $x^4 + x^8$, General Antiderivative: $\frac{1}{5}x^5 + \frac{1}{9}x^9 + C$

(3) What does the Net Change Theorem say? Write the formal statement then explain it in your own words.

Net Change Theorem: The integral of a rate of change is the net change.

In other words, the net change in a function is the (definite) integral of its derivative

(4) Give several examples of application problems where we could use the Net Change Theorem.

Q: Water is flowing into a tank at a rate of $r(t) = t^2 ft^3/min$. How much water flows into the tank over the time interval 1 min. to 5 min.?
S: Let $V = V(t)$ be the volume of the water in the tank. Then $V'(t) = r(t) = t^2$ so

$$V(5) - V(1) = \int_1^5 r(t)dt = \int_1^5 t^2 dt = \frac{t^3}{3}\Big|_1^5$$
$$= \frac{5^3}{3} - \frac{1^3}{3} = \frac{125 - 1}{3} = \frac{124}{3}$$

Extra Practice in Book: 5.4: 1, 7, 9, 11, 21, 29, 49, 51, 53, 56, 64,