Section 4.7: Optimization

(1) In this section, we learn how to use calculus to find the minimum and maximum value of functions in modeling problems. We have used calculus before to find the minimum and maximum values of a function. Describe how this works.

In Section 4.1, we learned the Closed Interval Method to find the absolute maximum and minimum values of a continuous function on a closed interval. To apply this method, we find the function values at the critical numbers and at the endpoints. The largest of these value is the absolute maximum and the smallest is the absolute minimum.
In Section 4.3, we learned the First Derivative Test and the Second Derivative Test to determine the local maximum and minimum of a continuous function. Both of these tests involve first finding the critical numbers of a given function. Next, for the First Derivative Test we look at the sign changes of f' near a critical number and for Second Derivative Test we look at the sign of f'' at a critical number.

(2) When solving an optimization problem, there are several steps we often take. What are those steps?

1. Understand the problem: What is the unknown? What are the given quantities? What are the given conditions?
2. Draw a Diagram
3. Introduce Notation: Assign a symbol to the quantity that is to be maximized or minimized. Also select symbols for other unknown quantities and label the diagram with these symbols.
4. Express the quantity being maximized or minimized in terms of some of the other symbols from Step 3.
5. If the function has more than one variable use the given information to find relationships (in the form of equations) among these variables. Then use these equations to eliminate all but one of the variables in the function. Write the domain of this function in the given context.
6. Use the methods of Sections 4.1 and Sections 4.3 to find the absolute maximum or minimum value of the function.

(3) Once we find our critical points, we need to check that we actually have an absolute minimum or maximum. How can we do this?

If the domain of our function is a closed interval, then the Closed Interval Method in Section 4.1 can be used. Otherwise, we can use the First Derivative Test for Absolute Extreme Values from this section.

(4) Just like in the related rates section, there are various expressions/formulas/theorems you need to use when solving these problems. Make a note of the ones that come up in the examples you've done. Provide details for any that you are not completely comfortable with.

Volume of a cylinder:

$$V = \pi r^2 h,$$

Surface area of a cylinder:

$$A = 2\pi r^2 + 2\pi r h$$

Distance between two points (x_1, y_1) and (x_2, y_2):

$$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

Extra Practice in Book: 4.7: 1, 5, 7, 11, 16, 21, 36