Section 4.1: Maximum and Minimum Values

(1) In this section, we talk about minimum and maximum values. First, let's make sure we know the definitions. What is an absolute minimum or maximum value? What is a local maximum or minimum value?

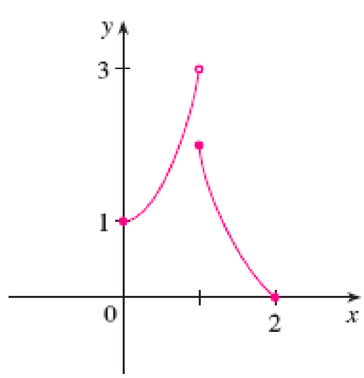
> Let c be a number in the domain D of the function f. Then $f(c)$ is the:
>
> Absolute maximum of f on D if $f(c) \geq f(x)$ for all x in D
> Absolute minimum of f on D if $f(c) \leq f(x)$ for all x in D
> Local maximum of f if $f(c) \geq f(x)$ when x is near c
> Local minimum of f if $f(c) \leq f(x)$ when x is near c

(2) What does the extreme value theorem say?

> If f is continuous on a closed interval $[a, b]$, then f attains an absolute maximum value $f(c)$ and an absolute minimum value $f(d)$ at some numbers c and d in $[a, b]$.

(3) In order for the extreme value theorem to apply you need to have a continuous function on a closed interval. Let's see what can go wrong if you don't.

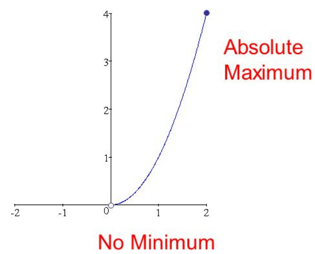
(a) Give an example of a function (a sketch of its graph is sufficient) which is not continuous on $[a, b]$ and does not have an absolute maximum but does have an absolute minimum.



(b) Give an example of a continuous function on $(a, b]$ (a sketch of its graph is sufficient) that does not have an absolute minimum but does have an absolute maximum.

$y = x^2$

$D = (0, 2]$



(4) What does Fermat's theorem say? How do we use it? What are critical numbers? How are they related to Fermat's theorem?

> Fermat's theorem says that if f has a local maximum or minimum at c, and if $f'(c)$ exists, then $f'(c) = 0$.
> A critical number of a function f is a number c in the domain of f such that either $f'(c) = 0$ or $f'(c)$ does not exist.
> So if f has a local maximum or minimum at c, then c is a critical number of f.

(5) (Important) What do we need to do to determine the absolute minimum and maximum value of a function?

> The closed interval method
> To find the absolute maximum and minimum values of a continuous function f on a closed interval $[a, b]$:
> 1. Find the values of f at the critical numbers of f in (a, b).
> 2. Find the values of f at the endpoints of the interval.
> 3. The largest of the values from Steps 1 and 2 is the absolute maximum value; the smallest of these values is the absolute minimum value.

Extra Practice in Book: 4.1: 1, 2, 3, 5, 7, 11, 49, 53, 55,