

Section 3.3: Derivatives of Trigonometric Functions

(1) In this section, we learn the derivatives of the 6 trig functions. Write out each of these functions and their derivatives.

(2) Using the derivatives of $\sin(x)$ and $\cos(x)$ and the quotient and chain rules, you can prove the derivative rules for the other four trig functions. Derive the derivative formula for $\sec(x)$ using this method.

- (3) Since $\pm \sin(x)$ and $\pm \cos(x)$ are each others derivatives, if we start taking higher order derivatives, we will notice a repeating pattern. Find a formula for the n th derivative of $\sin(x)$. (You will probably want to use a piecewise function depending on what the remainder is when you divide n by 4).

- (4) In this section, we learn that $\lim_{x \rightarrow 0} \frac{\sin(x)}{x} = 1$ and $\lim_{x \rightarrow 0} \frac{\cos(x) - 1}{x} = 0$. Using these limits, we can solve other limits involving trig functions. Explain how you would find the following limit, where m and n are real numbers.

$$\lim_{x \rightarrow 0} \frac{\sin(mx)}{nx}$$