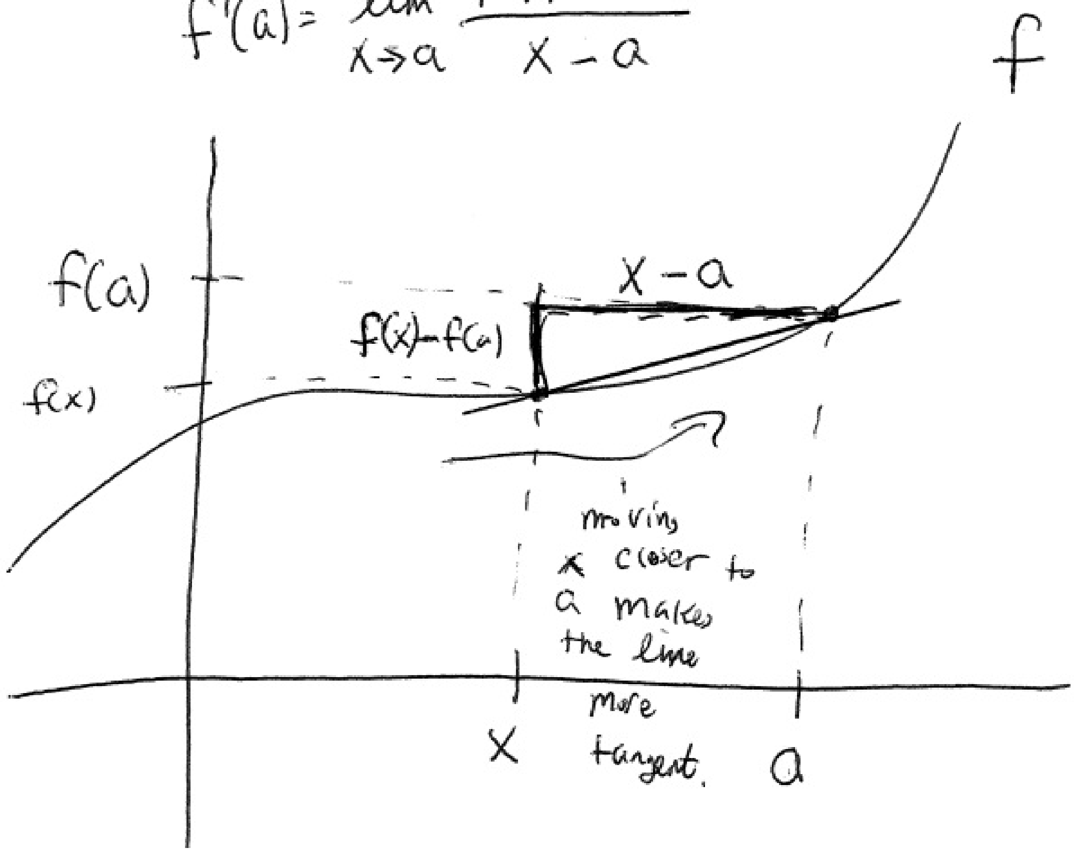
Section 2.7: Derivatives and Rates of Change

(1) In this section, we focus on finding the slope of the tangent line to a curve $f(x)$ at a point $x = a$. There are two different (but equivalent) limit definition we can use to do this. What is the limit definition that has both an x and an a in it? Draw a graph to illustrate how this definition works.

Answer: The limit definition with both an x and a in it is

$$f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}.$$

$$f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

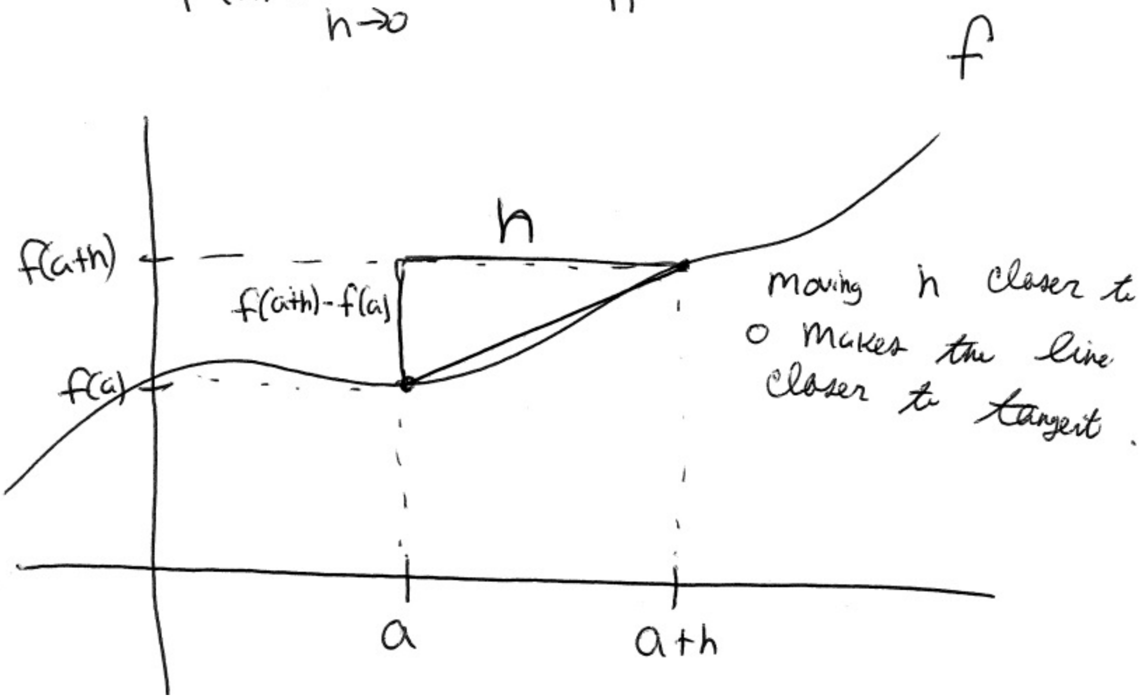


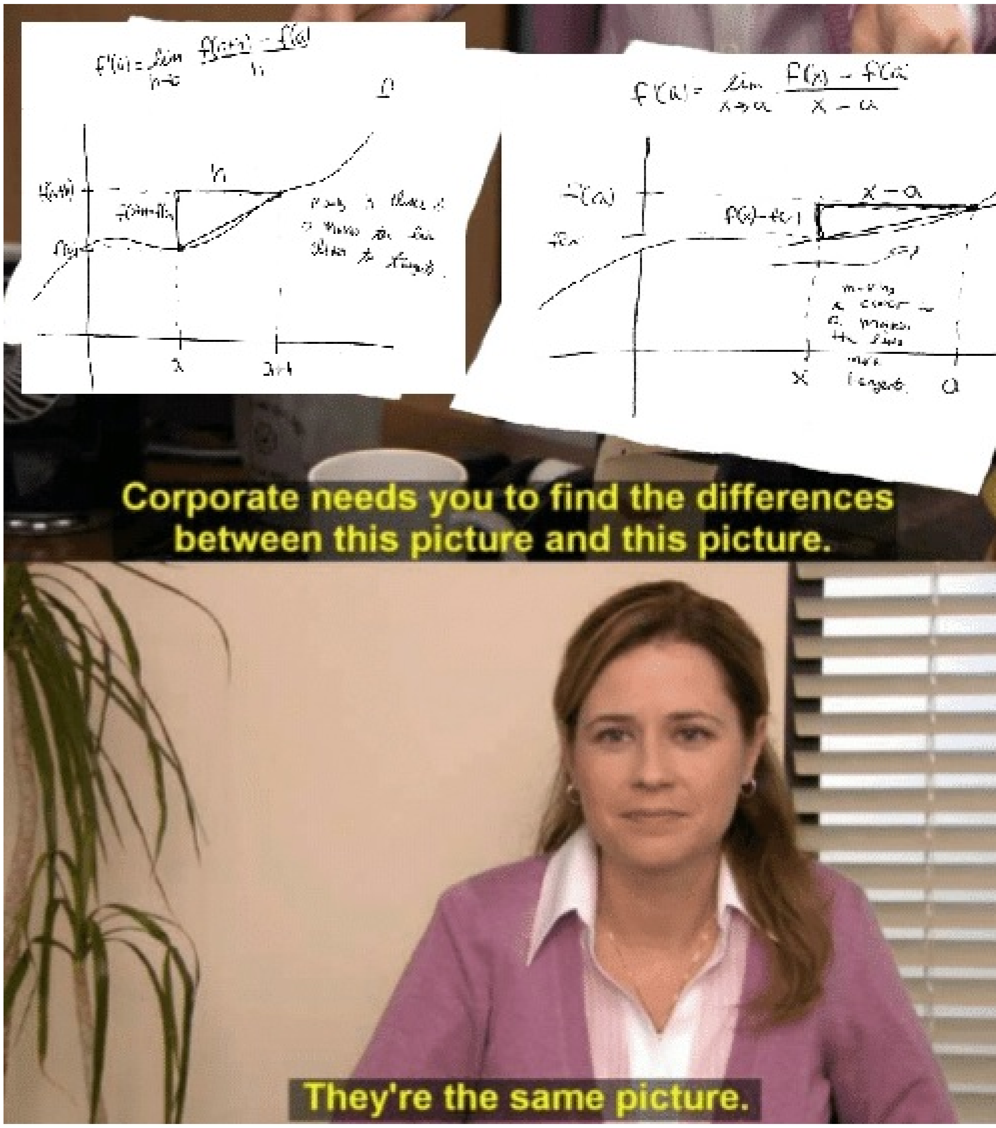
(2) What is the limit definition that has both an a and an h in it? Draw a graph to illustrate how this definition works.

Answer: The other definition that uses a and h is

$$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}.$$

$$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$





(3) What other term do we use for the slope of the tangent line of a curve $f(x)$ at $x = a$?

Answer: The derivative at a

(4) If our function is a position function, then what is another term for the slope of the tangent line or derivative at a?

Answer: The instantaneous velocity

(5) Write down the average rate of change for a function $f(x)$ on the interval from $[x, a]$ and also on the interval $[a, a+h]$. How do these compare to the formulas for the slope of the tangent lines. Relate instantaneous rate of change and slope of the tangent line and the derivative.

The average rate of change on $[x, a]$ is equal to the slope of the line between $(x, f(x))$ and $(a, f(a))$, also known as the secant line between those two points. This slope is given by

$$\frac{f(x) - f(a)}{x - a}.$$

Similarly for the interval $[a, a + h]$, the slope is given by

$$\frac{f(a + h) - f(a)}{h}.$$

The slope of the tangent line, a.k.a. the derivative, is given by taking the limits of these two expressions as x approaches a and as h approaches 0, respectively.

Formulas/Ideas to Know

$$\begin{aligned}
\text{Slope of the tangent line at } (x = a) &= \text{derivative at } x = a \\
&= f'(a) \\
&= \lim_{x \to a} \frac{f(x) - f(a)}{x - a} \\
&= \lim_{h \to 0} \frac{f(a + h) - f(a)}{h} \\
&= \text{instantaneous rate of change at } x = a \\
&= \text{instantaneous velocity at } x = a \text{ (if } f \text{ is a position function)}
\end{aligned}$$

Extra Practice in Book: 2.7: 7, 9, 11, 12, 15, 17, 35, 37, 53