Section 2.4: The Precise Definition of a Limit

(1) Explain the precise definition of a limit in your own words. What is the role of δ and of ε?

> The precise definiton of $\lim_{x\to a} f(x) = L$ is that we can make $f(x)$ as close as possbile (within ε for any $\varepsilon > 0$) by letting x get close to a (within δ).

(2) You will be asked to find a δ given a specific ε in both graph questions and given functions (usually linear). If you are given a specific value for ε you should get a specific value for δ. If you are asked to do it for a general ε, then your answer for δ will be in terms of ε. Let's practice that in an example. Let $f(x) = 2x + 1$. We will show that $\lim\limits_{x\to 3} f(x) = 7$

(a) Fill in the blanks:

For every $\underline{\varepsilon > 0}$ there exists $\underline{\delta > 0}$, such that $\underline{|f(x) - L|} < \varepsilon$ whenever $\underline{|x - a|} < \delta$.

(b) Let $\varepsilon = .5$. Find δ so that $|f(x) - 7| < \varepsilon$ whenever $|x - 3| < \delta$. Illustrate with a graph.

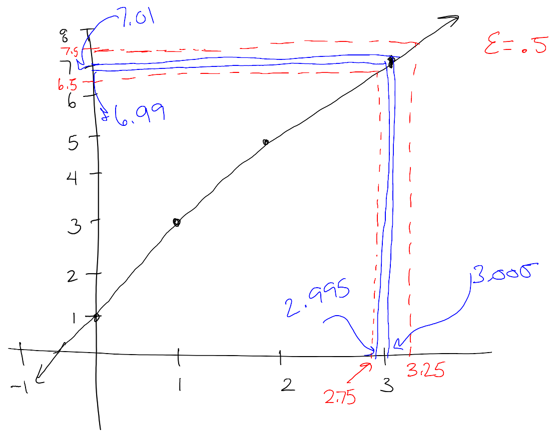
> We want $|f(x)-7| < \varepsilon = .5$, thus we want $|2x+1-7| = |2x-6| < .5$. Factoring out the 2, we get that $2|x - 3| < .5$. So dividing by 2, we get $|x - 3| < .25$. Thus, we can see that when $|x-3| < 1/4$, we can get $|f(x) - L| < \varepsilon$ so we get $\delta = 1/4$.

(c) Let $\varepsilon = .01$. Find δ so that $|f(x) - 7| < \varepsilon$ whenever $|x - 3| < \delta$. Illustrate with a graph.

> We want $|f(x) - 7| < \varepsilon = .01$, thus we want $|2x + 1 - 7| = |2x - 6| < .01$. Factoring out the 2, we get that $2|x - 3| < .01$. So dividing by 2, we get $|x - 3| < .005$. Thus, we can see that when $|x - 3| < .005$, we can get $|f(x) - L| < \varepsilon$ so we get $\delta = .005$.

(d) Find δ(in terms of ε) so that $|f(x) - 7| < \varepsilon$ whenever $|x - 3| < \delta$. Illustrate with a graph.

> We want $|f(x) - 7| < \varepsilon$, thus we want $|2x + 1 - 7| = |2x - 6| < \varepsilon$. Factoring out the 2, we get that $2|x - 3| < \varepsilon$. So dividing by 2, we get $|x - 3| < \varepsilon/2$. Thus, we can see that when $|x - 3| < \varepsilon/2$, we can get $|f(x) - L| < \varepsilon$ so we get $\delta = \varepsilon/2$.



Extra Practice in Book: 2.4:1, 3, 11, 15, 17