*University of Connecticut*
*Department of Mathematics*

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MATH 1131 PRACTICE EXAM 3

NAME: Solutions

SIGNATURE: ______________________

Instructor Name: ______________________ Lecture Section: ________

TA Name: ______________________ Discussion Section: ________

**Read This First!**

- Please read each question carefully. All questions are multiple choice. There is only one correct choice for each answer. Each question is one point.

- Indicate your answers on the answer sheet. The answer sheet is the **ONLY** place that counts as your official answers.

  (1) When you're done, hand in **both** the exam booklet and the answer sheet.

  (2) You will receive the exam booklet back after the exam is graded. The booklet is not graded, but you may circle answers there for your records.

- Calculators are allowed **below the level of TI-89**. In particular, tl **is not allowed**. No books or other references are permitted.

1. Which of the following is the absolute minimum value of the function $f(x) = \dfrac{x}{x^2+2}$ on the interval $[1, 3]$?

$\boxed{C}$

(A) $\frac{1}{4}$ (B) $\frac{1}{3}$ (C) $\frac{3}{11}$

(D) $\frac{\sqrt{2}}{4}$ (E) $\frac{\sqrt{2}}{\sqrt{2}+2}$

$$f'(x) = \frac{(x^2+2) - x(2x)}{(x^2+2)^2}$$

$$= \frac{-x^2+2}{(x^2+2)^2} = 0$$

never undefined

$x^2 = 2 \Rightarrow x = \pm\sqrt{2}$, $-\sqrt{2}$ not in interval

| $x$ | $f(x)$ |
|---|---|
| 1 | $1/3 \approx .33$ |
| $\sqrt{2}$ | $\sqrt{2}/4 \approx .35$ |
| 3 | $3/11 = .27$ |

min

2. Assume that a certain function $f(x)$ is continuous on the interval $[a, b]$ and differentiable on the open interval $(a, b)$, with $a < b$. If $f(a) = 3$, $b - a = 2$, and $f'(c) \geq 2.5$ for all $c$ with $a < c < b$, then use the Mean Value Theorem to determine the smallest possible value of $f(b)$. [1]

$\boxed{D}$

(A) 2 (B) 3 (C) $\frac{17}{4}$

(D) 8 (E) $\frac{37}{4}$

By MVT

$$\frac{f(b)-f(a)}{b-a} = f'(c) \geq 2.5$$

so $$\frac{f(b)-3}{2} \geq 2.5$$

$$f(b) - 3 \geq 5$$

$$f(b) \geq 8$$

3. Find all value(s) of $x$ where $f(x) = 2x^3 + 3x^2 - 12x$ has a local minimum. [1]

(A) 1 (B) $-2$ (C) $-2,\ 1$

(D) $-2,\ \frac{1}{2}$ (E) $-2,\ \frac{1}{2},\ 1$

$\boxed{A}$

$$f'(x) = 6x^2 + 6x - 12$$
$$6(x^2 + x - 2)$$
$$6(x-1)(x+2)$$
$$x = 1, 2$$

$f'$: $+ + +$ | $- - - -$ | $+ + + +$ ; $-2$, $1$

local max @ $x = -2$
local min @ $x = 1$

4. The number of points at which $f(x) = x^4 - 8x^2 - 7$ has an inflection point is which of the following?

(A) 0 (B) 1 (C) 2

(D) 3 (E) 4

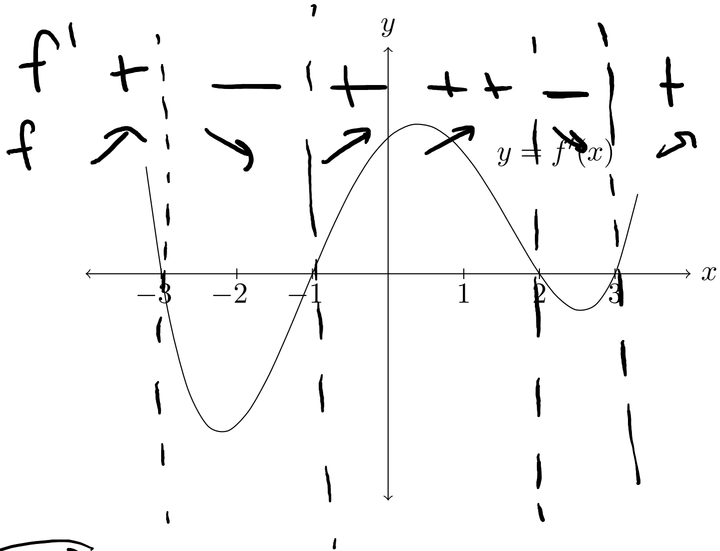
$\boxed{C}$

$$f'(x) = 4x^3 - 16x$$
$$f''(x) = 12x^2 - 16$$
$$x^2 = \frac{16}{12}$$
$$x = \pm\frac{4}{\sqrt{12}} = \pm\frac{2}{\sqrt{3}}$$

$f''$: $+ + +$ ⊛ $- - -$ ⊛ $+ + +$ ; $-2/\sqrt{3}$, $\frac{2}{\sqrt{3}}$

2 POI at ⊛'s

5. Below is the graph of the *derivative* $f'(x)$ of a function $f(x)$. At what $x$-value(s) does $f(x)$ have a local maximum or local minimum?




max @ $-3, 2$

min @ $-1, 3$

$\boxed{A}$

(A) Local maxima at $-3$ and 2 and local minima at $-1$ and 3

(B) Local maxima at $-1$ and 3 and local minima at $-3$ and 2

(C) Local maxima at $-1$ and 3 and local minimum at 2

(D) Local maxima at $-3$ and 2 and local minimum at $-1$

(E) None of the above

6. Referring to the same graph of the derivative in question 5, at approximately what $x$-value(s) is $f(x)$ concave up?

$\boxed{C}$

f c.u. when $f'' > 0$ when $f'$ increasing

(A) $x < -1$ and $x > 1.5$

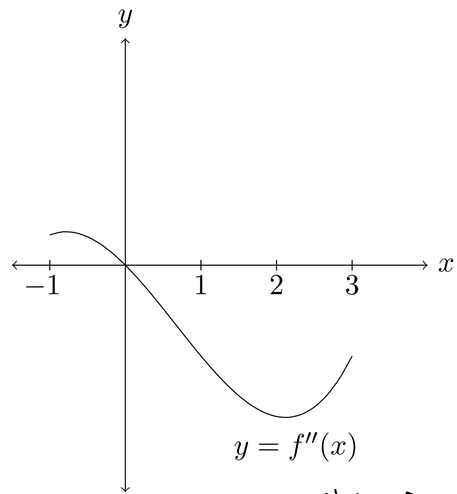
(B) $-1 < x < 2$

(C) $-2.1 < x < .8$ and $x > 2.6$

(D) $-\infty < x < \infty$

(E) We cannot determine concavity of $f(x)$ from the graph of $f'(x)$.

7. Below is the graph of the *second derivative* $f''(x)$ of a function $f(x)$ on the interval $[-1, 3]$. Which of the following statements must be true?




$f'' > 0$ ✓

$\boxed{E}$

(A) The function $f(x)$ is concave up when $-1 < x < 0$.

(B) The derivative is decreasing when $0 < x < 3$. $f'$ dec $\Rightarrow f'' < 0$ ✓

(C) The function has a point of inflection at $x = 0$. $f''$ change $+$ to $-$ ✓

(D) The derivative $f'(x)$ has a local maximum at $x = 0$.

(E) All of the above

$f'$ inc to dec ✓
$\Rightarrow f''$ $\oplus$ to $\ominus$

8. On which interval(s) is the function $f(x) = x^4 - 6x^3 + 12x^2 + 1$ concave down?

(A) $(-\infty, 1)$ only (B) $(1, 2)$ only (C) $(-\infty, -1)$ and $(2, \infty)$

(D) $(2, \infty)$ only (E) $(-\infty, 1)$ and $(2, \infty)$

$\boxed{B}$

$f'(x) = 4x^3 - 18x^2 + 24x$

$f''(x) = 12x^2 - 36x + 24$

$12(x^2 - 3x + 2)$

$f''$: $++$ | $- - - -$ | $+++$ ; cd between 1 and 2



$(x-1)(x-2)$

$x = 1, 2$

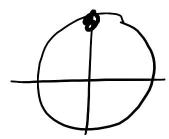
9. Evaluate the following limit:

$$\lim_{x\to 0^+} \frac{\sin x}{x^2}. = \frac{0}{0} = \lim_{x\to 0^+} \frac{\cos x}{2x} = \frac{1}{+0}$$

(A) $+\infty$ (B) $-\infty$ (C) 0

(D) 1/2 (E) $-1/2$

$\boxed{A}$



10. Evaluate the following limit:

$$\lim_{x\to \frac{\pi}{2}} \frac{1-\sin x}{\cos x}. = \frac{0}{0} = \lim_{x\to \pi/2} \frac{-\cos x}{-\sin x} \quad \frac{0}{1}$$

(A) 0 (B) 1 (C) $+\infty$

(D) $-1$ (E) 1/2

$\boxed{A}$

D

11. Determine the number of inflection points of the graph of $y = \tan(x)$ in the interval $\left(-\frac{3\pi}{2}, \frac{3\pi}{2}\right)$. [1]

(A) 0 (B) 1 (C) 2 (D) 3 (E) 5

$y = \tan x$ , undefined $\pm \pi/2$

$y' = \sec^2 x$

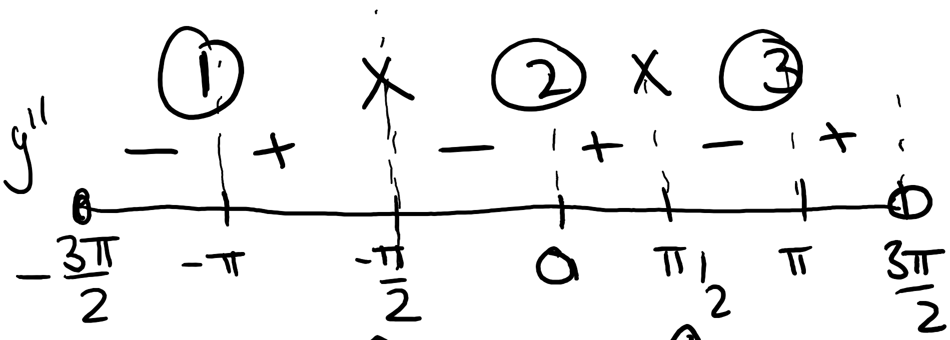
$y'' = 2\sec x \cdot \sec x \cdot \tan x$

$= 2\sec^2 x \tan x$

$= \dfrac{\sin x}{\cos^3 x}$

$= 0$ $\sin x = 0$ $0, \pi, -\pi$

undefined $\pm \pi/2$




NOT POI since function undefined

3 POIs

12. Find two positive numbers $x$ and $y$ satisfying $y + 2x = 80$ whose product is a maximum. [1]

(A) 24, 32 (B) 26, 28 (C) 20, 40

(D) 26, 27 (E) None of the above

C

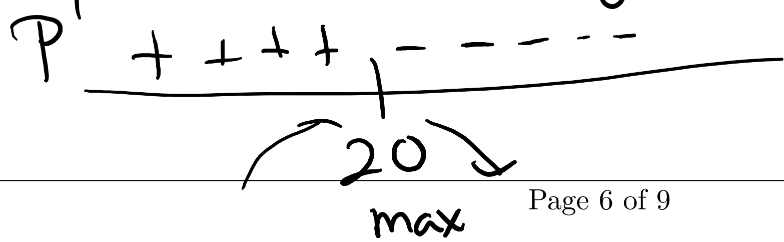
Max : $xy$ $y = 80 - 2x$

$P = x(80 - 2x)$

$= 80x - 2x^2$

$P' = 80 - 4x$

$x = 20, y = 40$



$\boxed{C}$

13. A certain function $f(x)$ satisfies $f''(x) = 2 - 3x$ with $f'(0) = -1$ and $f(0) = 1$. Compute $f(2)$.

(A) $-3$ (B) $-2$ (C) $-1$

(D) 1 (E) 3

$f'(x) = 2x - \frac{3x^2}{2} + C \quad f'(0) = C = -1$

$f'(x) = 2x - \frac{3x^2}{2} - 1$

$f(x) = x^2 - \frac{x^3}{2} - x + C$

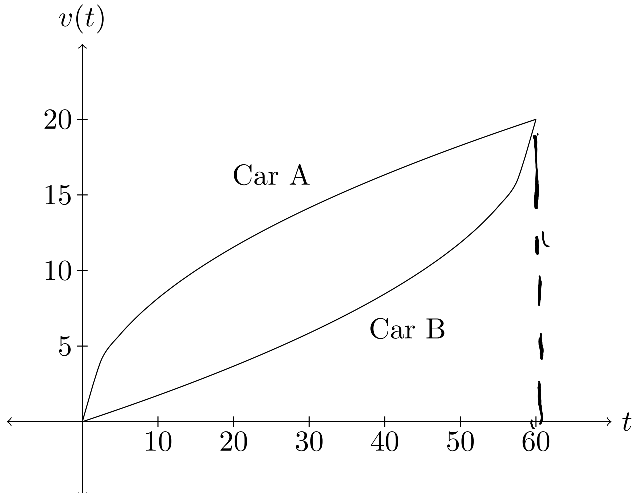
$f(0) = 0 = 1$

$f(x) = x^2 - \frac{x^3}{2} - x + 1$

$f(2) = 4 - 4 - 2 + 1 = -1$

14. Below is the graph of the velocity (measured in ft/sec) over the interval $0 \le t \le 60$ for two cars, Car A and Car B. How do the distances traveled by each compare at $t = 60$?

$\boxed{A}$




distance traveled = area under curve

more area under curve of Car A

(A) Car A has traveled further than Car B ✓

(B) Car B has traveled further than Car A ✗

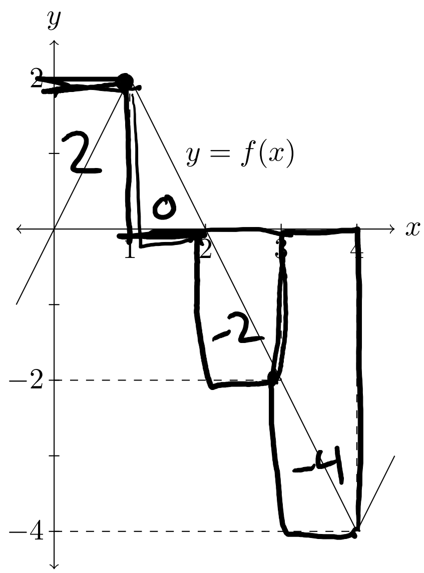
(C) Car A and Car B have traveled the same distance ✗

(D) Cannot be determined because we don't know their position functions ✗

(E) Cannot be determined because we don't know the equations of their velocity curves ✗

15. If we use a right endpoint approximation with four subintervals (i.e., $R_4$), then what is the resulting approximation for [1]

$$\int_0^4 f(x)\,dx?$$




$\boxed{B}$

(A) 2 (B) −4 (C) −2

(D) 0 (E) −1

$2(1) + 0(1) + (-2)(1) + -4(1)$

16. Evaluate the definite integral $\int_{-1}^{1} (x^2 + 2x + 1)\,dx$.

(A) 8/3 (B) −1 (C) 5/3

(D) −5/3 (E) 0

$\boxed{A}$

$$= \frac{x^3}{3} + x^2 + x \Big|_{-1}^{1}$$

$$\left(\tfrac{1}{3} + 1 + 1\right) - \left(\tfrac{-1}{3} + 1 - 1\right)$$



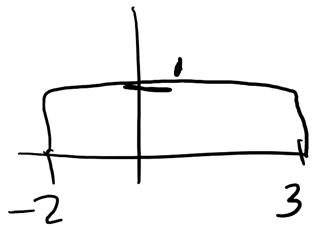
$7/3 + 1/3 = 8/3$

17. Assume that $\int_{-2}^{3} f(x)\,dx = 4$. What is the value of $\int_{-2}^{3} (f(x)+1)\,dx$?

$\boxed{D}$

(A) 4 (B) 5 (C) 6

$\boxed{\text{(D) } 9}$ (E) 20




$$\int_{-2}^{3} f(x)\,dx + \int_{-2}^{3} 1\,dx$$

$$4 + 5(1) = 9$$

18. Which of the following is the correct derivative of the function [1]

$$f(x) = \int_{1}^{x^2} \frac{1}{t^3+1}\,dt?$$

(A) $\frac{2x}{x^6+1}$ (B) $\frac{1}{x^6+1}$ (C) $\frac{2x}{x^5+1}$

(D) $\frac{1}{x^3+1}$ (E) $\frac{2x}{x^3+1}$

$\boxed{A}$

$$= \frac{1}{(x^2)^3+1}\frac{d}{dx}(x^2)$$

$$= \frac{2x}{x^6+1}$$