University of Connecticut
Department of Mathematics

MATH 1131 | PRACTICE EXAM 2 | FALL 2018

NAME: Solutions.

SIGNATURE: ______________________

Instructor Name: ______________________ Lecture Section: __________

TA Name: ______________________ Discussion Section: __________

Read This First!

- Please read each question carefully. All questions are multiple choice. There is only one correct choice for each answer. Each question is one point.
- Indicate your answers on the answer sheet. The answer sheet is the **ONLY** place that counts as your official answers.

  (1) When you're done, hand in **both** the exam booklet and the answer sheet.

  (2) You will receive the exam booklet back after the exam is graded. The booklet is not graded, but you may circle answers there for your records.

- Calculators are allowed **below the level of TI-89**. In particular, the **TI-Nspire is not allowed**. No books or other references are permitted.

1. Determine $f'(1)$ for the function $f(x) = (x^3 - x^2 + 1)(x^4 - x + 2)$.

(A) 3 (B) 0 (C) 4

(D) 2 (E) 5

$$f'(x) = (x^3 - x^2 + 1)(4x^3 - 1) + (3x^2 - 2x)(x^4 - x + 2)$$

$$f'(1) = (1 - 1 + 1)(4 - 1) + (3 - 2)(1 - 1 + 2)$$

$$= (1)(3) + (1)(2) = 5$$

2. Find the equation of the tangent line to the curve $y = \dfrac{x}{x+1}$ at $x = 1$.

(A) $y = \frac{1}{2}$ (B) $y = -\frac{1}{2}x + 1$ (C) $y = \frac{1}{2}x$

(D) $y = -\frac{1}{4}x + \frac{3}{4}$ (E) $y = \frac{1}{4}x + \frac{1}{4}$

$$y' = \frac{(x+1) \cdot 1 - x \cdot (1)}{(x+1)^2} \qquad y'(x=1) = \frac{2-1}{(2)^2} = \frac{1}{4}$$

$$y(x=1) = \frac{1}{1+1} = \frac{1}{2}$$

pt $(1, 1/2)$ $m = \frac{1}{4}$

$$y - \frac{1}{2} = \frac{1}{4}(x - 1)$$

$$y = \frac{1}{4}x - \frac{1}{4} + \frac{1}{2} \Rightarrow y = \frac{1}{4}x + \frac{1}{4}$$

3. If $f(x) = \sin(x)$, determine $f^{(125)}(\pi)$.

(A) 1 (B) -1 (C) 0

(D) 1/2 (E) $\sqrt{2}/2$

$f'(x) = \cos x$

$f''(x) = -\sin x$

$f'''(x) = -\cos x$

$f^{(4)}(x) = -(-\sin x)$
$= \sin x$

Recognise the pattern on ~~how~~ the derivatives are being repeated

So $125 = 4 \cdot (31) + 1$

$f^{(125)}(x) = \cos x$

$f^{(125)}(\pi) = \cos \pi = -1$

4. To compute the derivative of $\sin^2 x$ with the chain rule by writing this function as a composition $f(g(x))$, what is the "inner" function $g(x)$? [1]

(A) x (B) x^2 (C) $\sin x$

(D) $\sin^2 x$ (E) None of the above

$f(g(x)) = (\sin x)^2 = \sin^2 x$

$\frac{d}{dx}(\sin x)^2 = 2 \cdot \sin x \cdot \frac{d}{dx} \sin x$

$= 2 \cdot \sin x \cdot \cos x$

5. Let $y = f(x)g(x)$. Using the table of values below, determine the value of $\frac{dy}{dx}$ when $x = 2$. [1]

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	5	2	4	4
2	3	4	1	3
3	2	3	2	2
4	4	1	5	5
5	1	5	3	1

(A) 9 (B) 12 (C) 13

(D) 15 (E) 23

$y = f(x) \cdot g(x)$

$y' = f'(x) \cdot g(x) + f(x) \cdot g'(x)$

$y'\big|_{x=2} = f'(2) \cdot g(2) + f(2) \cdot g'(2) = 4 \cdot 1 + 3 \cdot 3$

$= 4 + 9 = 13.$

6. For the function $f(x) = 2\sin x - 3\cos x$ determine $f'''(\pi/2)$, its third derivative at $x = \pi/2$. [1]

(A) 2 (B) -2 (C) 3

(D) -3 (E) 1

$f(x) = 2\sin x - 3\cos x$

$f'(x) = 2\cos x - 3(-\sin x)$

$f''(x) = -2\sin x + 3\cos x$

$f'''(x) = -2\cos x - 3\sin x$

$f'''(x = \pi/2) = -2\cos \pi/2 - 3\sin(\pi/2) = -3(1) = -3$

7. If $g(x) = \dfrac{ax+b}{cx+d}$, where a, b, c, and d are constants, then $g'(1)$ is which of the following? [1]

(A) $\dfrac{a+b-c-d}{c+d}$ (B) $\dfrac{ad-bc}{(c+d)^2}$ (C) $\dfrac{a+b-c-d}{(c+d)^2}$

(D) $\dfrac{ad+bc}{c+d}$ (E) $\dfrac{ad+bc}{(c+d)^2}$

$$g'(x) = \frac{(cx+d).(a) - (ax+b)(c)}{(cx+d)^2}$$

$$g'(1) = \frac{(c+d)a-(a+b)c}{(c+d)^2}$$

$$= \frac{\cancel{c \cdot a} + a \cdot d - \cancel{a \cdot c} - c \cdot b}{(c+d)^2}$$

$$= \frac{ad-bc}{(c+d)^2}$$

8. For the function $f(x) = x^3 \arctan(x)$, which of the following is $f'(1)$?

(A) $\dfrac{3\pi}{4}$ (B) $\dfrac{3\pi}{4} + \dfrac{1}{2}$ (C) $\dfrac{1}{2}$

(D) $\dfrac{\pi}{4}$ (E) $3\tan(1) + \sec^2(1)$

$$f'(x) = 3x^2 \cdot \arctan(x) + x^3 \cdot \frac{1}{1+x^2}$$

$$f'(1) = 3(1) \cdot \arctan(1) + 1^3 \cdot \frac{1}{1+1^2}$$

$$= 3 \cdot (1) \cdot \frac{(\pi)}{4} + \frac{1}{2}$$

$$= \frac{3\pi}{4} + \frac{1}{2}$$

9. On the curve $x^y = y^x$ with x and y both positive, $\frac{dy}{dx}$ is which of the following? [1]

(A) $\frac{1-\ln x}{1-\ln y}$ (B) $x^{y-x}\ln x$ (C) $(1-\ln x)\frac{y}{x}$

(D) $(1-\ln y)\frac{y}{x}$ (E) $\left(\frac{y}{x}\right)\left(\frac{x\ln y - y}{y\ln x - x}\right)$

$$x^y = y^x$$
$$\ln x^y = \ln y^x$$
$$y\ln x = x\cdot\ln y$$
$$\left(y\cdot\frac{1}{x} + y'\cdot\ln x\right) = 1\cdot\ln y + x\cdot\frac{1}{y}\cdot y'$$
$$y'\ln x - \frac{x}{y}y' = \ln y - \frac{y}{x}$$
$$y'\left(\ln x - \frac{x}{y}\right) = \left(\ln y - \frac{y}{x}\right)$$
$$y' = \frac{\left(\ln y - \frac{y}{x}\right)}{\left(\ln x - \frac{x}{y}\right)} = \frac{\frac{x\ln y - y}{x}}{\frac{y\ln x - x}{y}}$$
$$= \frac{(x\ln y - y)}{(y\ln x - x)}\cdot\frac{y}{x}$$

10. Find $\frac{d}{dx}\left[x^{\ln x}\right]$. [1]

(A) $(\ln x)x^{\ln x}$ (B) $2(\ln x)x^{(\ln x)+1}$ (C) $x^{(\ln x)-1}$

(D) $2(\ln x)x^{\ln x}$ (E) $2(\ln x)x^{(\ln x)-1}$

$$y = x^{\ln x}$$
$$\ln y = \ln x\cdot\ln x$$
$$\ln y = (\ln x)^2$$
$$\frac{1}{y}y' = 2\ln x\cdot\frac{1}{x}$$
$$y' = \frac{2\ln x}{x}\cdot y$$
$$y' = \frac{2\ln x}{x}\cdot x^{\ln x}$$
$$= 2\ln x\cdot x^{(\ln x)-1}$$

11. On the curve $xy^3 = x - y$, which of the following is $\frac{dy}{dx}$?

(A) $\frac{1-y^2}{1+2xy^2}$ (B) $\frac{1-y^3}{1-3xy^2}$ (C) $\frac{1+y^3}{1+3xy^2}$

(D) $\frac{1+y^2}{1+3xy^2}$ (E) $\frac{1-y^3}{1+3xy^2}$

$$x \cdot 3y^2 \cdot y' + 1 \cdot y^3 = 1 - y'$$

$$(3xy^2 + 1)y' = 1 - y^3$$

$$y' = \frac{1-y^3}{(3xy^2+1)}$$

12. The size of a colony of bacteria at time t hours is given by $P(t) = 100e^{kt}$, where P is measured in millions. If $P(5) > P(0)$, then determine which of the following is true. [1]

I. $k > 0$

II. $P'(5) < 0$

III. $P'(10) = 100ke^{10k}$

(A) I and III only. (B) I and II only. (C) I only.

(D) II only. (E) I, II, and III.

$P(t) = 100e^{kt}$ is an exponential model, that is a solution to differential Eqn $\frac{dP}{dt} = k \cdot P$ with $P(0) = 100$

If $P(5) > P(0)$, then the population is growing so $k > 0$ $\Rightarrow$ I true.

$P'(t) = 100k \cdot e^{kt}$; $P'(5) = 100ke^{5k} > 0$ $\Rightarrow$ II False

$P'(10) = 100ke^{10k}$ $\Rightarrow$ III True

13. Suppose that the half-life of a certain substance is 20 days and there are initially 10 grams of it. The amount of the substance, in grams, remaining after t days is [1]

(A) $10e^{10k}$ (B) $\ln(10)e^{kt/10}$ (C) $\ln(10)e^{t/10}$

(D) $10e^{-t\ln(2)/20}$ (E) $10e^{t\ln(2)/20}$

Let $A(t)$ be amount of substance @ any time 't'

$A(t)=10$ $A(20)=5$

$t=0$ $t=20$ day

$A(t)=ce^{kt}$

$10=c\cdot e^{k0}$

$\Rightarrow c=10$

$A(t)=10e^{kt}$

$A(20)=10e^{k\cdot 20}=5$

$\frac{1}{2}=e^{k\cdot 20}$

$\Rightarrow \frac{\ln(1/2)}{20}=k$

$\frac{\ln(1)-\ln 2}{20}=k$

$\Rightarrow k=\frac{-\ln 2}{20}$

$A(t)=10e^{-\frac{\ln(2)}{20}\cdot t}$

14. Atmospheric pressure (the pressure of air around you) goes down as your height above sea level goes up. It decreases exponentially by 12% for every 1000 meters of added height. The pressure at sea level is 101.3 kilopascals. The pressure in kilopascals at height h meters above sea level is given by [1]

(A) $1000e^{10h}$ (B) $\ln(101.3)e^{kh/12}$ (C) $101.3e^{\ln(0.88)/1000}$

(D) $1000e^{-h\ln(2)/20}$ (E) $101.3e^{h\ln(0.88)/1000}$

$p(h)=101.3$ $p(1000)=101.3-(0.12)101.3$
$=89.144$

$h=0$ (sealevel) $h=1000$

$p(h)=101.3\,e^{\frac{\ln(0.88)}{1000}\cdot h}$

$p(h)=P_0\,e^{k\cdot h}$

$p(1000)=89.144=101.3\,e^{k\cdot 1000}$

$0.88=e^{k1000}$ $k=\frac{\ln(0.88)}{1000}$

15. A particle moves along the curve $y^3 = x^4 + 11$. When it reaches the point $(2, 3)$, the y-coordinate is increasing at a rate of 32 cm/s. Which of the following represents the rate of increase of the x-coordinate at that instant?

(A) 27 cm/s (B) 9 cm/s (C) 13.5 cm/s

(D) 6.75 cm/s (E) None of the above

$\frac{dy}{dt} = 32 \text{cm/s}$

$x = 2$

$y = 3$

$y^3 = x^4 + 11$

$3y^2 \frac{dy}{dt} = 4x^3 \cdot \frac{dx}{dt}$

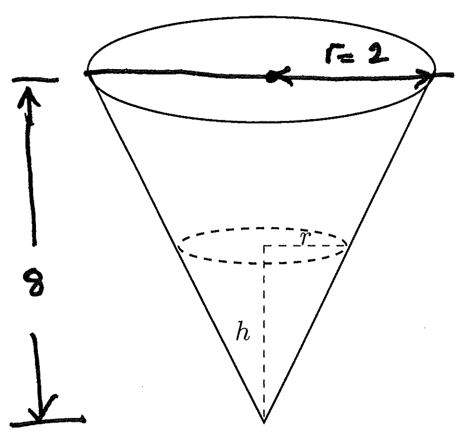
$3(3)^2 \cdot 32 = 4(2)^3 \cdot \frac{dx}{dt}$

$\frac{dx}{dt} = 27 \text{cm/s}$

16. Water is withdrawn at a constant rate of 2 ft^3/min from an inverted cone-shaped tank (meaning the vertex is at the bottom). The diameter of the top of the tank is 4 ft, and the height of the tank is 8 ft. How fast is the water level falling when the height of the water in the tank is 2 ft? (**Note**: the volume of a cone of height h and base a circle of radius r is $\frac{1}{3}\pi r^2 h$.)

(A) $\frac{2}{\pi}$ ft/min (B) $\frac{4}{\pi}$ ft/min (C) $\frac{6}{\pi}$ ft/min

(D) $\frac{8}{\pi}$ ft/min (E) $\frac{16}{\pi}$ ft/min



$\frac{dv}{dt} = 2 \text{ ft}^3/\text{min}$

$\frac{dh}{dt} = ?$ when $h = 2$

$\frac{2}{8} = \frac{r}{h}$ $r = \frac{1}{4}h$

$V = \frac{1}{3}\pi r^2 h$

$= \frac{1}{3}\pi \left(\frac{1}{4}h\right)^2 \cdot h = \frac{\pi}{48} h^3$

$V = \frac{\pi}{48} h^3$

$\frac{dV}{dt} = \frac{\pi}{48} \cdot 3h^2 \cdot \frac{dh}{dt}$

$2 = \frac{\pi}{\cancel{48}\,\cancel{16}\,4} \cancel{3}(\cancel{2})^{\cancel{2}} \cdot \frac{dh}{dt}$

$\frac{dh}{dt} = \frac{8}{\pi}$ ft/min

17. Use the linearization for the function $f(x) = \sqrt{x^3 + 2x + 1}$ at $x = 1$ to approximate the value of $f(1.1)$.

(A) 2.0125 (B) 2.10 (C) 2.125

(D) 0.5 (E) 1.925

$f(x) = (x^3+2x+1)^{1/2}$ $f(1) = 2$

$f'(x) = \frac{1}{2\sqrt{(x^3+2x+1)}}(3x^2+2)$ $f'(1) = \frac{5}{4}$

$L(x) = f(a) + f'(a)(x-a)$

$L(x) = 2 + \frac{5}{4}(x-1)$

$L(1.1) = 2 + \frac{5}{4}(1.1-1) = 2.125$

18. Let $f(x) = x^2 - 10$. If $x_1 = 3$ in Newton's method for solving $f(x) = 0$, determine x_2.

(A) 1/2 (B) 19/6 (C) 15/4

(D) 12/7 (E) 17/6

$f(x) = x^2 - 10$

$f'(x) = 2x$

$x_1 = 3$

$f(x_1) = f(3) = 3^2 - 10 = 1$

$f'(x_1) = f'(3) = 2 \cdot 3 = 6$

$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$

$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$

$x_2 = 3 - \frac{(-1)}{6}$

$x_2 = 19/6$