
The Fundamental Theorem of Calculus

Solutions should show all of your work, not just a single final answer.

1. In worksheet 5.1 you used rectangles to estimate the area under the curve $y = 4 - x^2$ over the interval $[0, 2]$. Use the Fundamental Theorem of Calculus to compute the exact area and then determine if the midpoint approximation from problem 1c in worksheet 5.1 is an underestimate or an overestimate of that area.

2. Use Part 1 of the Fundamental Theorem of Calculus to compute $f'(x)$ in each case below.

(a) $f(x) = \int_1^x \frac{1}{t^4 + 1} dt$

(b) $f(x) = \int_x^1 \cos \sqrt{t} dt$

(c) $f(x) = \int_1^{3x} \ln t dt$

(d) $f(x) = \int_0^{x^2} t \sin t dt$

3. Evaluate the following definite integrals using the Fundamental Theorem of Calculus.

(a) $\int_2^3 (x^3 + x) dx$

(b) $\int_{-1}^1 x^{20} dx$

(c) $\int_2^{10} \frac{1}{\sqrt{x}} dx$

(d) $\int_0^1 e^{x+1} dx$

4. (a) Let $A_0(x) = \int_0^x (1 - t^2) dt$, $A_1(x) = \int_1^x (1 - t^2) dt$, and $A_2(x) = \int_2^x (1 - t^2) dt$.

Compute these explicitly in terms of x using Part 2 of the Fundamental Theorem of Calculus.

- (b) Over the interval $[0, 2]$, use your answers in part (a) to sketch the graphs of $y = A_0(x)$, $y = A_1(x)$, and $y = A_2(x)$ on the same set of axes.

- (c) How are the three graphs in part (a) related to each other? In particular, what does Part 1 of the Fundamental Theorem of Calculus tell you about the graphs in part (a)?

- (d) On a graph of $y = 1 - t^2$, for $0 \leq t \leq 2$, shade the region with signed area $A_0(1.5)$. Indicate with $+$ and $-$ which area counts positively and which negatively.

5. T/F (with justification) The function $F(x) = \int_0^x \cos(t^2) dt$ is an antiderivative of $\cos(x^2)$.

6. T/F (with justification) $\int_{-2}^2 x^{-4} dx = \frac{x^{-3}}{-3} \Big|_{-2}^2 = -\frac{1}{12}$.