Newton's Method

Solutions should show all of your work, not just a single final answer.

1. Apply Newton's method to estimate the solution of $x^3 - x - 1 = 0$ by taking $x_1 = 1$ and finding the least n such that x_n and x_{n+1} agree to three digits after the decimal point.

2. The number π is a solution of $\sin x = 0$ close to 3 (see Figure 1). You will use Newton's method for $\sin x = 0$ to create numerical estimates for π.

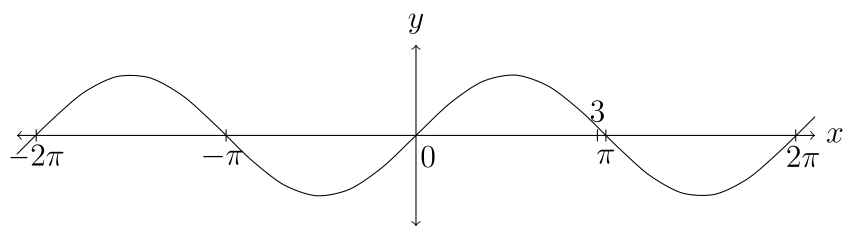

Figure 1: Graph of $y = \sin x$.

   (a) Write out the recursion for Newton's method used to solve $\sin x = 0$.

   (b) Using Newton's method for $\sin x = 0$ with $x_1 = 3$, find the first n for which x_n and x_{n+1} agree to 5 digits after the decimal point. (Use radians, *not* degrees!)

   (c) For the n you found in part (b), to how many digits after the decimal point does x_n actually agree with π?

3. In Figure 2 is the graph of $f(x) = \ln(x) - 1$ for $0 < x < 4$. It crosses the x-axis at $x = e$. You will use Newton's method for $f(x) = 0$ to create numerical estimates for e.

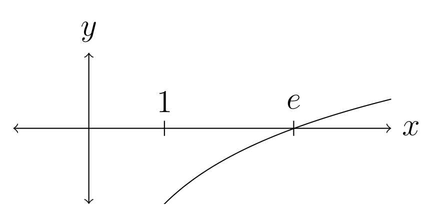

Figure 2: Graph of $y = \ln(x) - 1$.

   (a) Using Newton's method for the equation $\ln(x) - 1 = 0$ with $x_1 = 1$, tabulate x_n to find the first n for which x_n and x_{n+1} agree to 5 digits after the decimal point.

   (b) For the n you found in part (a), to how many digits after the decimal point does x_n actually agree with e?