How Derivatives Affect the Shape of a Graph

Solutions should show all of your work, not just a single final answer.

1. For the following functions, (i) determine all open intervals where $f(x)$ is increasing, decreasing, concave up, and concave down, and (ii) find all local maxima, local minima, and inflection points. Give all answers **exactly**, not as numerical approximations.

   (a) $f(x) = x^5 - 2x^3$ for all x

   (b) $f(x) = x + \sin x$ for $-2\pi < x < 2\pi$

   (c) $f(x) = e^{-x} - e^{-3x}$ for $x > 0$

2. For x in the interval $(0, 100)$, let $f(x) = x^{100} + (100 - x)^{100}$. Determine on what open intervals in $(0, 100)$ the function $f(x)$ is increasing and decreasing, and use this information to decide which of $33^{100} + 67^{100}$ or $41^{100} + 59^{100}$ is larger.

3. T/F (with justification) If a function $f(x)$ on the interval $(-1, 1)$ is twice differentiable and $f''(c) = 0$ for some c in $(-1, 1)$ then $f(x)$ has an inflection point at $x = c$.