Exponential Growth and Decay

Solutions should show all of your work, not just a single final answer.

1. In 1859, 24 rabbits were released into the wild in Australia, where they had no natural predators. Their population grew exponentially, doubling every 6 months.
   (a) Determine $P(t)$, the function that gives the population at time t, **and** the differential equation describing the population growth. Let units for t be years.
   (b) After how many years, rounded to one digit after the decimal point, did the rabbit population reach 1,000,000?
   (c) Determine the *rate* of population change, in rabbits/year, midway through the third year. (**Warning**: t is not 3.5, just like the year midway through the 21st century is not 2150.) Round the final answer to 2 digits after the decimal point.

2. The element Unobtainium has a half-life of 3 years. Let $M(t)$ be the mass of Unobtainium at time t starting with an initial amount of 14 kg.
   (a) Give a formula for $M(t)$.
   (b) After how many years will the initial mass of Unobtainium shrink to 1 kg? Round your answer to one digit after the decimal point.

3. Starbucks serves coffee at 170° and the room temperature in Starbucks is 70°. The coffee cools to 100° after 10 minutes. Let $T(t)$ be the temperature of the coffee at time t, measured in minutes.
   (a) Write down the differential equation for $T(t)$ and determine a formula for $T(t)$.
   (b) From the time when the temperature is 100° at $t = 10$, how many *additional* minutes will it take for the temperature of the coffee to reach 80°? Round your answer to one digit after the decimal point.

4. T/F (with justification) If $\dfrac{dy}{dx} = y$ then $y = 0$ or $y = e^x$.

5. T/F (with justification) A function $y(t)$ satisfying $\dfrac{dy}{dt} = -.01y$ has $\lim_{t\to\infty} y(t) = 0$.