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# Limits at Infinity and Horizontal Asymptotes

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**Solutions should show all of your work, not just a single final answer.**

1. Find the limit in each case or explain why it does not exist (and if it is  $\pm\infty$ ).

(a)  $\lim_{x \rightarrow \infty} \frac{2x + 3}{2x - 7}$

(b)  $\lim_{x \rightarrow \infty} \frac{100000x}{x^3 + x}$

(c)  $\lim_{x \rightarrow \infty} \frac{x^3}{\sqrt{6x^4 - 1}}$

(d)  $\lim_{x \rightarrow \infty} \frac{\sqrt{16x^4 + 7x}}{8x^2 + 5}$

(e)  $\lim_{x \rightarrow \infty} \sqrt{x^2 + 2x} - x$

(f)  $\lim_{x \rightarrow \infty} \frac{e^{3x} - e^{-3x}}{e^{3x} + e^{-3x}}$

2. Let  $f(x) = \frac{\sqrt{4x^6 + 5}}{x^3 - 1}$ .

(a) Compute  $\lim_{x \rightarrow \infty} f(x)$ .

(b) Compute  $\lim_{x \rightarrow -\infty} f(x)$ .

(c) What are the horizontal asymptotes of the graph of  $y = f(x)$ ?

(d) What is the vertical asymptote of the graph of  $y = f(x)$ ?

3. T/F (with justification) The graph of the function  $y(x) = 3 + 6e^{-kx}$ , with  $k$  a positive constant, has a horizontal asymptote  $y = 6$ .
4. T/F (with justification) If the continuous function  $f(x)$  has domain  $(-\infty, +\infty)$ , then either  $\lim_{x \rightarrow \infty} f(x)$  exists or  $\lim_{x \rightarrow \infty} f(x)$  is  $\pm\infty$ .