Continuity

Solutions should show all of your work, not just a single final answer.

1. Let
$$f(x) = \begin{cases} x^2 + x & \text{if } x < 1, \\ a & \text{if } x = 1, \\ x - 1 & \text{if } x > 1. \end{cases}$$
   (a) Determine the value of a for which $f(x)$ is continuous from the left at 1.
   (b) Determine the value of a for which $f(x)$ is continuous from the right at 1.
   (c) Is there a value of a for which $f(x)$ is continuous at 1? Explain.

2. Let
$$f(x) = \begin{cases} 2 - kx & \text{if } x < 1, \\ k + x & \text{if } x > 1 \end{cases}$$
   with the value of $f(1)$ to be determined.
   (a) Compute $\lim_{x \to 1^-} f(x)$ in terms of k.
   (b) Compute $\lim_{x \to 1^+} f(x)$ in terms of k.
   (c) Find the values of k and $f(1)$ that make $f(x)$ continuous at $x = 1$.
   (d) Using the choice of k and $f(1)$ in part (c), make a graph of $y = f(x)$ for $0 \leq x \leq 2$.

3. T/F (with justification) The function
$$f(x) = \begin{cases} \sin x & \text{if } x \leq 0, \\ 1 + \cos x & \text{if } x > 0 \end{cases}$$
   has a jump discontinuity at $x = 0$.

4. T/F (with justification) A function that is continuous at a point has to be defined at the point.

5. T/F (with justification) A function that is discontinuous at a point can't be defined at the point.