# Calculating Limits Using the Limit Laws

**Solutions should show all of your work, not just a single final answer.**

1. State the limit laws, including for powers and roots, and the direct substitution property.

2. Let
$$f(x) = \begin{cases} x^2 + 1 & \text{if } x < 1, \\ 4 & \text{if } x = 1, \\ x + 2 & \text{if } 1 < x \leq 2, \\ 6 - x & \text{if } x > 2. \end{cases}$$

   (a) Sketch the graph of $y = f(x)$ for $-1 \leq x \leq 4$.

   (b) Evaluate the following limits if they exist. (If a limit does not exist, write DNE.)

   (i) $\lim_{x \to 1^-} f(x)$ (ii) $\lim_{x \to 1^+} f(x)$ (iii) $\lim_{x \to 1} f(x)$

   (iv) $\lim_{x \to 2^-} f(x)$ (v) $\lim_{x \to 2^+} f(x)$ (vi) $\lim_{x \to 2} f(x)$

3. Evaluate the following limits exactly using algebra and limit laws (some limits may be DNE).

   (a) $\lim_{x \to 2} \dfrac{x^3 - 2}{2x^2 - 3x + 2}$

   (b) $\lim_{x \to -2} \sqrt{x^4 + 3x + 6}$

   (c) $\lim_{x \to 9} \dfrac{x - 9}{\sqrt{x} - 3}$

   (d) $\lim_{x \to 1} \dfrac{x^2 + 4x}{x^2 + 3x - 4}$

   (e) $\lim_{x \to 3} \dfrac{\sqrt{x^2 + 40} - 7}{x - 3}$

   (f) $\lim_{x \to 1} \dfrac{(x^2 + x)^2 - 4}{x^2 + x - 2}$

4. Evaluate the following limits using algebra and limit laws (some limits may be DNE). Note that $a$ represents a constant, and answers may be in terms of $a$.

   (a) $\lim_{t \to 0} \dfrac{\sqrt{a + t} - \sqrt{a - t}}{t}$ for $a > 0$

   (b) $\lim_{h \to 0} \dfrac{1/(a + h)^2 - 1/a^2}{h}$ for $a \neq 0$

5. T/F (with justification) If $\lim_{x \to 2} g(x) = 0$ and $\lim_{x \to 2} h(x) = 0$ then $\lim_{x \to 2} \dfrac{g(x)}{h(x)}$ does not exist.