Algebra Review

In your solutions to the exercises below, **show the algebraic work** that leads to the final answer.

Simplifying Algebraic Expressions

1. Simplify the expression $\dfrac{1}{3^{-2}} - \dfrac{1}{3} + \dfrac{1}{4^{-1}}$.

2. Simplify the expression $\dfrac{(x^2y^{-3})^2}{(y^{-3}x^{-2})^{-2}}$.

3. Simplify $(4x^6)^{3/2}$.

4. If $f(x) = x^2 + 3x$ and $h \neq 0$, then simplify $\dfrac{f(x+h) - f(x)}{h}$.

5. Simplify $\dfrac{1}{x+h} - \dfrac{1}{x}$.

6. Rationalize $\dfrac{3}{x - \sqrt{x}}$.

Intervals

7. Write in interval notation
   (a) the open interval with endpoints 2 and 3,
   (b) the closed interval with endpoints 2 and 3,
   (c) the half-open interval with endpoints 2 and 3 that contains 2 but not 3.

8. Represent the following sets of numbers in two ways: interval notation and drawn as subsets of the number line:

   (a) $1 \leq x < 3$, (b) $-2 \leq x \leq 2$, (c) $-5 < x \leq -3$.

Completing the Square

9. Complete the square for the following expressions:

   (a) $x^2 - 8x + 12$, (b) $s^2 + 3s - 6$.

10. Write the following expressions as a difference of squares:

    (a) $x^2 + 4x$, (b) $y^2 + 5y$.

Solving Equations

11. Solve for x in terms of y: $2y^2x - y^2 - (1 + 3y) = x$.

12. Find the solutions of $\dfrac{x^2}{3} + 2x - 1 = 0$ exactly (that is, not approximations).

13. Find the solutions of $\dfrac{1}{x-4} + \dfrac{1}{x+4} = \dfrac{4}{x^2-16}$ exactly.

14. Find the solutions of $\dfrac{1}{x} - \dfrac{1}{x+2} = 2$ exactly.

Factoring

15. Factor $x^2 - 2x - 24$.

16. Factor as much as possible $x^3 - a^2x$.

Exponential and Logarithmic Functions

17. Simplify

    (a) $\dfrac{2^{5x}}{2^x}$ (b) $e^{2x}e^{-3x}$ (c) $\dfrac{e^{2x}-1}{e^x-1}$ (d) $\sqrt[3]{5^{2x}}$

18. Evaluate $\log_4(1/64)$.

19. Solve for x exactly: (a) $\log_2 x + \log_2(x-2) = 3$ and (b) $\ln x - \ln(x^2) = 5$.

20. Find all $x > 0$ that satisfy $x^{\sqrt{x}} = x\sqrt{x}$. (Hint: write everything in terms of $y = \sqrt{x}$.) Check your solution(s) work.

Trigonometric Functions

21. On the unit circle mark off the following angles (in radians):

    (a) $\dfrac{\pi}{2}, \pi,$ and $-\dfrac{\pi}{2}$ together (b) $\dfrac{\pi}{3}$ and $\dfrac{2\pi}{3}$ together.

22. Evaluate the following, where k is an integer in part d.

    (a) $\sin\left(\dfrac{7\pi}{2}\right)$ (b) $\cos\left(\dfrac{-\pi}{2}\right)$ (c) $\sin\Big(101\pi\Big)$ (d) $\sin\left(\dfrac{\pi}{2} + 2k\pi\right)$.

Inverse Functions

23. Find the inverse of each of the following functions, including the domain, if an inverse exists.

$$\text{(a) } f(x) = \frac{x}{1+2x} \text{ for } x \neq -\frac{1}{2} \qquad \text{(b) } f(x) = \sqrt{18-2x^2} \text{ for } 0 \leq x \leq 3.$$

24. Find the inverse of each of the following functions, including the domain, if an inverse exists.

$$\text{(a) } f(x) = \ln(e^{2x}+1) \text{ for all } x \qquad \text{(b) } f(x) = \frac{e^x}{1+2e^x} \text{ for all } x.$$

Graphs

25. Below is a graph of $y = f(x)$. Sketch the following graphs, using a new set of axes each time. Pay attention to translation, compression, and stretching of the graph.

(a) $y = f(x+1)$ (b) $y = f(x-1)$ (c) $y = f(2x)$ (d) $y = 2f(x)$.

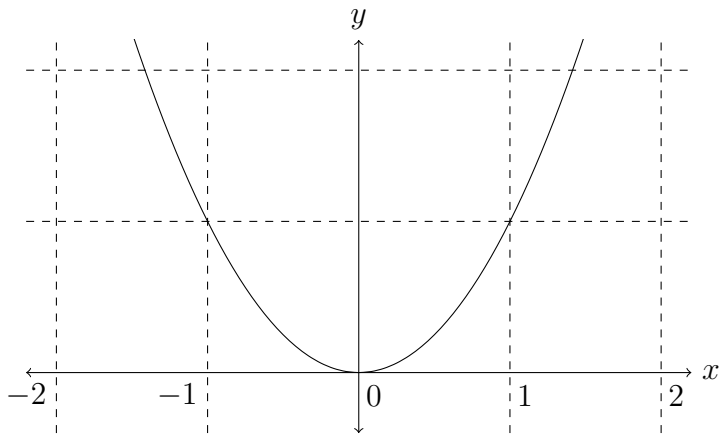

Figure 1: Graph of $y = f(x)$.