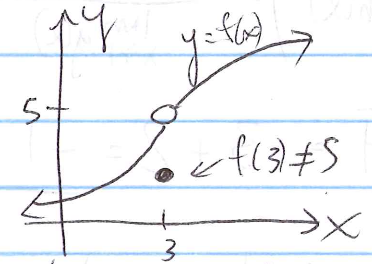
Practice Exam 1 Solutions

(A) 1. $V_{avg} = \frac{s(4)-s(1)}{4-1} = \frac{(16+12)-(1+3)}{3} = \frac{24}{3} = \underline{8}$

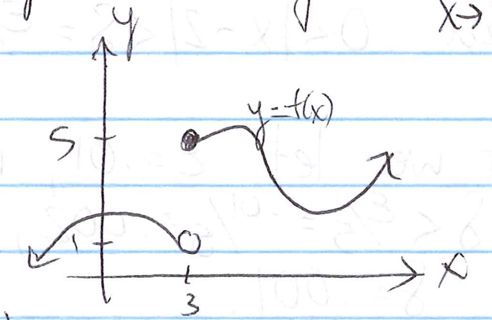
(D) 2. $\lim_{x \to 1^-} \frac{x-3}{x-1}$. For $x = 0.9999$, $x-3<0$ and $x-1<0$,

so $\lim_{x\to 1^-} \frac{x-3}{x-1} = \underline{+\infty}$.

(E) 3. If $\lim_{x\to 3^+} f(x) = 5$, then several possibilities exist for $\lim_{x\to 3^-} f(x)$. $f(3)$ may not be defined, and even if it is, $\lim_{x\to 3^-} f(x)$ need not be $f(3)$:



There could also be, for example, a jump discontinuity and maybe $\lim_{x\to 3^-} f(x) = 1$:



Therefore, $\lim_{x\to 3^-} f(x)$ $\underline{\text{cannot be determined}}$.

(B) 4. $-x^2-x+1 \le g(x) \le x^2-x+1$ for all $x \ne 0$, so

$$\underbrace{\lim_{x\to 0}(-x^2-x+1)}_{=1} \le \lim_{x\to 0} g(x) \le \underbrace{\lim_{x\to 0}(x^2-x+1)}_{=1}$$

Therefore, we must have $\lim_{x\to 0} g(x) = \underline{1}$.

(A) 5. $\lim_{x\to 4} \frac{x^2-8x+16}{x-4} = \lim_{x\to 4} \frac{\cancel{(x-4)}(x-4)}{\cancel{x-4}} = \lim_{x\to 4} x-4 = \underline{0}$.

(Note: $\frac{x^2-8x+16}{x-4} = x-4$ only if $x \ne 4$!)

(A) 6. $\lim_{x\to 1}\left(\frac{2f(x)}{g(x)} + \sqrt{h(x)}\right) = \frac{2\lim_{x\to 1} f(x)}{\lim_{x\to 1} g(x)} + \sqrt{\lim_{x\to 1} h(x)}$

$= \frac{2\cdot 3}{-2} + \sqrt{4} = -3+2 = \underline{-1}$.

(E) 7. This line (which is the graph of $f(x)=5x+1$) has slope 5, so if we want $|f(x)-11| < \varepsilon$, we will need $0 < |x-2| < \delta = \varepsilon/5$.

Therefore, if we let $\varepsilon = .01$, we need to choose $\delta < \varepsilon/5 = .01/5 = .002$, so we can choose $\underline{\delta = .001}$.

(A) 8. For <u>any</u> value of k, $f(x)$ is continuous on the intervals $(-\infty, 1)$ and $(1, \infty)$. To be continuous at $x=1$, we need

$$\lim_{x\to 1} f(x) = f(1) = k+1.$$

$$\lim_{x\to 1^-} f(x) = 1-k \text{ and } \lim_{x\to 1^+} f(x) = k+1,$$

so we need $1-k = k+1 \Rightarrow 2k=0 \Rightarrow \underline{k=0}$.

(C) 9. I. $\lim_{x\to 1^+} h(x) = 1$ and exists ✓

II. $\lim_{x\to 1^-} h(x) = \frac{1}{1} = 1$ and exists ✓

III. $\lim_{x\to 1^-} h(x) = \lim_{x\to 1^+} h(x) = 1$, so $\lim_{x\to 1} h(x) = 1$ and exists ✓

IV. $h(1)$ is undefined, so $\lim_{x\to 1} h(x) \neq h(1)$ ✗

Therefore, I, II, and III only.

(E) 10. $f(x)$ is guaranteed by the Intermediate Value Theorem to take all values between 1 and 11 since it is continuous on the interval $[-1, 3]$.

3, $\sqrt{2}$ (≈ 1.414), and 3π (≈ 9.425) are all between 1 and 11, so I, II, and III.

(B) 11. $x^3+8=0 \Rightarrow x^3=-8 \Rightarrow x=-2$

x^2+1 is not 0 when $x=-2$, so $\underline{x=-2 \text{ is a vertical asymptote}}$

$$\lim_{x\to\pm\infty} \frac{x^2+1}{x^3+8} = \lim_{x\to\pm\infty} \frac{\frac{1}{x}+\frac{1}{x^3}}{1+\frac{8}{x^3}} = \frac{0}{1} = 0, \text{ so}$$

$\underline{y=0 \text{ is horizontal asymptote}}$.

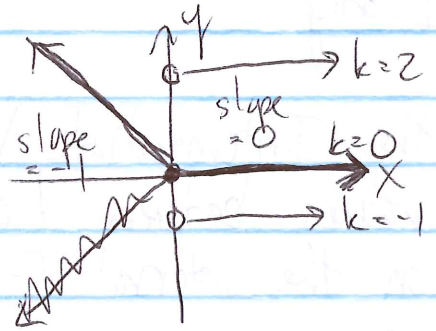
So there is $\underline{\text{one vertical and one horizontal asymptote}}$.

(D) 12. $$\lim_{x\to\infty} \frac{\sqrt{x^2+2}}{x} = \lim_{x\to\infty} \frac{\frac{1}{x}\sqrt{x^2+2}}{\frac{1}{x}\cdot x} = \lim_{x\to\infty} \frac{\sqrt{\frac{x^2+2}{x^2}}}{1} = \lim_{x\to\infty} \sqrt{1+\frac{2}{x^2}} = \underline{1}.$$

(D) 13. $\lim_{h\to 0} \frac{f(1+h)-f(1)}{h}$ represents $f'(1)$. For $f(x)=3x^{10}$,

we have $f'(x)=30x^9$, so $f'(1)=\underline{30}$.

(E) 14.



$\underline{\text{No value of } k \text{ makes this function differentiable at } x=0}$.
However, $f(x)$ would be continuous at $x=0$ if $k=0$.

(C) 15. $f(x)=\sqrt{x}+\frac{1}{\sqrt{x}}=x^{\frac{1}{2}}+x^{-\frac{1}{2}} \Rightarrow f'(x)=\frac{1}{2}x^{-\frac{1}{2}}-\frac{1}{2}x^{-\frac{3}{2}}$.

So $f'(4)=\frac{1}{2}4^{-\frac{1}{2}}-\frac{1}{2}4^{-\frac{3}{2}}=\frac{1}{2\sqrt{4}}-\frac{1}{2\sqrt{64}}=\frac{1}{4}-\frac{1}{16}=\underline{\frac{3}{16}}$.