The Fundamental Theorem of Calculus

Solutions should show all of your work, not just a single final answer.

1. State the two parts of the Fundamental Theorem of Calculus.

2. Use Part 1 of the Fundamental Theorem of Calculus to find the derivative of the function.

   (a) $f(x) = \displaystyle\int_1^x \frac{1}{t^3+1}\,dt$

   (b) $f(x) = \displaystyle\int_1^{e^x} \ln t\,dt$

   (c) $f(x) = \displaystyle\int_x^1 \cos\sqrt{t}\,dt$

   (d) $f(x) = \displaystyle\int_{1-2x}^{1+2x} t\sin t\,dt$

3. Evaluate the integral.

   (a) $\displaystyle\int_{-1}^{2} (x^3-2x)\,dx$

   (b) $\displaystyle\int_{-1}^{1} x^{100}\,dx$

   (c) $\displaystyle\int_1^{18} \sqrt{\frac{3}{x}}\,dx$

   (d) $\displaystyle\int_{-1}^{1} e^{x+1}\,dx$

4. (a) Let $A_0(x) = \displaystyle\int_0^x (1-t^2)\,dt$, $A_1(x) = \displaystyle\int_1^x (1-t^2)\,dt$, and $A_2(x) = \displaystyle\int_2^x (1-t^2)\,dt$. Compute these explicitly in terms of x using Part 2 of the Fundamental Theorem of Calculus.

   (b) Over the interval $[0,2]$, use your answers in part (a) to sketch the graphs of $y = A_0(x)$, $y = A_1(x)$, and $y = A_2(x)$ on the same set of axes.

   (c) How are the three graphs in part (a) related to each other? In particular, what does Part 1 of the Fundamental Theorem of Calculus tell you about the graphs in part (a)?

   (d) On a graph of $y = 1-t^2$, for $0 \le t \le 2$, shade the region with signed area $A_0(1.5)$. Indicate with $+$ and $-$ which area counts positively and which negatively.

5. Use a definite integral to create a function $g(x)$ such that $g'(x) = \cos(x^3)$ and $g(1) = 0$. Explain why your answer fits the required conditions.

6. T/F (with justification) $\displaystyle\int_{-4}^{2} x^{-3}\,dx = \frac{x^{-2}}{-2}\bigg|_{-4}^{2} = -\frac{2}{32}$.

7. T/F (with justification) $\displaystyle\frac{d}{dx}\left(\int_1^{10} \frac{1}{t}\,dt\right) = \frac{1}{10}$