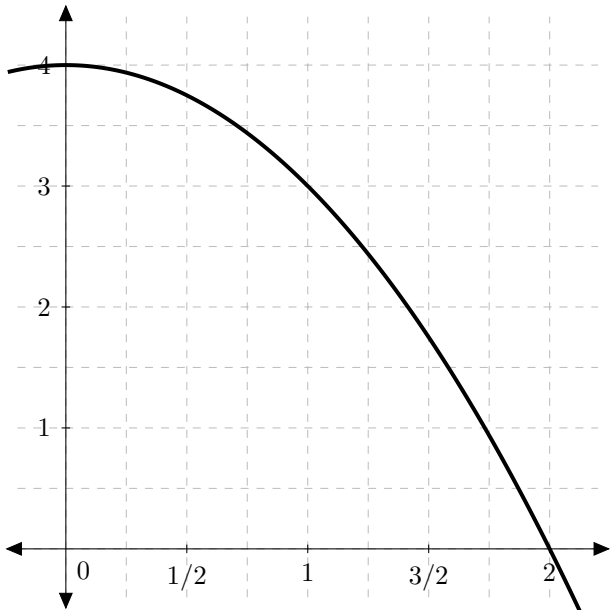
Areas and Distances / The Definite Integral

Solutions should show all of your work, not just a single final answer.

1. The graph of $y = 4 - x^2$ over the interval $[0, 2]$ is given below.



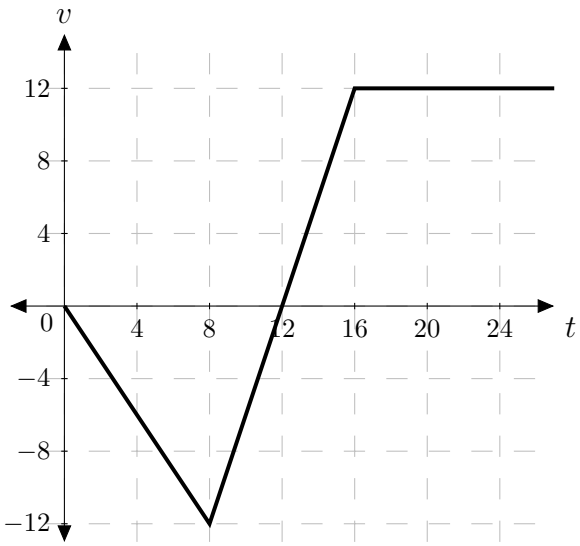
   (a) Estimate the area under the graph over $[0, 2]$ using 4 rectangles and right endpoints: sketch the rectangles and then compute their areas. Determine from your picture if the areas of the rectangles provide an underestimate or overestimate of the area under the curve, or if it can not be easily determined.

   (b) Repeat part (a) using 4 rectangles and left endpoints. Determine from your picture if the areas of the rectangles provide an underestimate or overestimate of the area under the curve, or if it can not be easily determined.

   (c) Repeat part (a) using 4 rectangles and midpoints. Determine from your picture if the areas of the rectangles provide an underestimate or overestimate of the area under the curve, or if it can not be easily determined.

Worksheet continues on next page

2. Here's a graph of the velocity (in ft/sec) of an object that is moving along a horizontal line with the right direction being positive and the left direction being negative.



(a) Describe the motion of the object over the interval $0 \leq t \leq 24$: when is it moving left or right, and when is it speeding up or slowing down?

(b) Compute the net change in the position of the object from $t = 0$ to $t = 8$ seconds, in feet. (A positive answer means a net change to the right, a negative answer means a net change to the left, and the answer 0 means the object ends up back where it began.)

(c) Compute the net change in position of the object between $t = 8$ and $t = 20$ seconds, in feet.

3. Evaluate the integral by intepreting it in terms of areas.

(a) $\int_{-1}^{1} (1 - x)dx$

(b) $\int_{-3}^{3} (1 + \sqrt{9 - x^2})dx$