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# Newton's Method

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**Solutions should show all of your work, not just a single final answer.**

1. Starting with  $x_1 = 2$ , find the third approximation  $x_3$  to the root of  $x^3 - 3x - 3 = 0$ . Round your answer to 3 decimal places.
2. Starting with  $x_1 = 1$ , apply Newton's method to solve  $x^3 - x - 1 = 0$  by finding the least  $n$  such that  $x_n$  and  $x_{n+1}$  agree to three decimal places.
3. The solutions to  $\sin x = 0$  are  $x = \pi k$ , where  $k$  is any whole number (see Figure 1). In particular, the smallest positive solution to  $\sin x = 0$  is  $x = \pi$ .

You will use Newton's method for  $\sin x = 0$  to make numerical estimates for  $\pi$ .

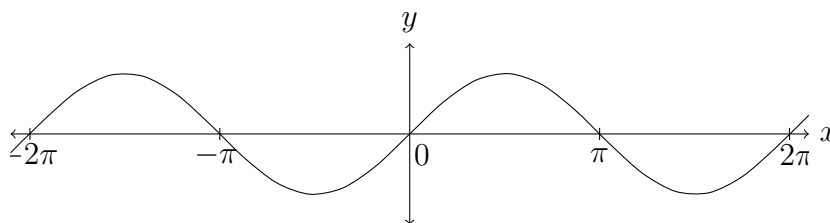


Figure 1: Graph of  $y = \sin x$ .

- (a) Use Newton's method for  $\sin x = 0$  with  $x_1 = 3$ , find the first  $x_n$  for which  $x_n$  and  $x_{n+1}$  agree to 5 digits decimal places. (Be sure you compute trigonometric functions using radians, *not* degrees!)
- (b) For the  $x_n$  you found in part (a), to how many decimal places does  $x_n$  actually agree with  $\pi$ ?