

How Derivatives Affect the Shape of a Graph

Solutions should show all of your work, not just a single final answer.

1. For the following functions, (i) determine all open intervals where $f(x)$ is increasing, decreasing, concave up, and concave down, and (ii) find all local maxima, local minima, and inflection points. Give all answers **exactly**, not as numerical approximations.

(a) $f(x) = x^5 - 2x^3$

(b) $f(x) = x + \sin x$ for $-2\pi \leq x \leq 2\pi$

(c) $f(x) = e^{-x} - e^{-3x}$

2. Let $f(x) = x^{100} + (100 - x)^{100}$. Determine where $f(x)$ is increasing and decreasing for $0 \leq x \leq 100$ and use this information to decide which of $33^{100} + 67^{100}$ or $41^{100} + 59^{100}$ is larger.
3. T/F (with justification) If $f''(a) = 0$ then $f(x)$ has an inflection point at $x = a$.