

Maximum and Minimum Values

Solutions should show all of your work, not just a single final answer.

1. For the following functions, find all critical points.

(a) $f(x) = x^5 - 2x^3$

(b) $f(x) = x + \sin x$ for $-2\pi \leq x \leq 2\pi$

(c) $f(x) = e^{-x} - e^{-3x}$

2. Find the absolute maximum and minimum values of the following functions in the given interval. Show your supporting work.

(a) $f(x) = 3x^4 - 4x^3 - 12x^2 + 12$ on $[-1, 3]$

(b) $f(x) = x^4 - 2x^2 + 3$ on $[-2, 3]$

(c) $f(x) = (7x - 1)e^{-2x}$ on $[0, 1]$

3. The cross-section of a 7-mile bike path is modeled by the graph of the function $f(x) = \frac{3}{50}(\frac{1}{4}x^4 - 3x^3 + \frac{15}{2}x^2 + 9x - 10)$ over the interval $[0, 7]$ (note that x and $f(x)$ are both measured in miles). For safety reasons, the slope of the course can never be greater than 1 or less than -1 . Find the maximum and minimum values of the slope of this path, and show that it satisfies the safety regulations.

4. T/F (with justification) A local maximum value of a function is also an absolute maximum value.

5. T/F (with justification) If $f'(a) = 0$, then $f(x)$ has a local maximum or minimum at $x = a$.