
Limits at Infinity and Horizontal Asymptotes

Solutions should show all of your work, not just a single final answer.

1. Find the limit in each case or explain why it does not exist (and if it is $\pm\infty$).

(a) $\lim_{x \rightarrow \infty} \frac{2x + 3}{2x - 7}$

(b) $\lim_{x \rightarrow \infty} \frac{100000x}{x^3 + x}$

(c) $\lim_{x \rightarrow \infty} \frac{x^3}{\sqrt{6x^4 - 1}}$

(d) $\lim_{x \rightarrow \infty} \frac{\sqrt{16x^4 + 7x}}{8x^2 + 5}$

(e) $\lim_{x \rightarrow \infty} \sqrt{x^2 + 2x} - x$

(f) $\lim_{x \rightarrow \infty} \frac{e^{3x} - e^{-3x}}{e^{3x} + e^{-3x}}$

2. Let $f(x) = \frac{\sqrt{4x^6 + 5}}{x^3 - 1}$.

(a) Compute $\lim_{x \rightarrow \infty} f(x)$.

(b) Compute $\lim_{x \rightarrow -\infty} f(x)$.

(c) What are the horizontal asymptotes of the graph $y = f(x)$?

(d) What is the vertical asymptote of the graph $y = f(x)$?

3. T/F (with justification) The function $y(t) = 3 + 6e^{-kt}$, with k a positive constant, has a horizontal asymptote $y = 6$.

4. T/F (with justification) If the continuous function $f(x)$ has domain $(-\infty, +\infty)$, then either $\lim_{x \rightarrow \infty} f(x)$ exists, or $\lim_{x \rightarrow \infty} f(x)$ is $\pm\infty$.