Continuity

Solutions should show all of your work, not just a single final answer.

1. Determine the exact value of the constant a making the following function continuous at 1:
$$f(x) = \begin{cases} \dfrac{x^2+2x-3}{x^2+5x-6} & \text{if } x \neq 1, \\ a & \text{if } x = 1. \end{cases}$$

2. Let
$$g(x) = \begin{cases} x^2 + x & \text{if } x < 1, \\ a & \text{if } x = 1, \\ x - 1 & \text{if } x > 1. \end{cases}$$
   (a) Determine the value of a for which g is continuous from the left at 1.
   (b) Determine the value of a for which g is continuous from the right at 1.
   (c) Is there a value of a for which g is continuous at 1? Explain.

3. Let
$$f(x) = \begin{cases} 2 - kx & \text{if } x < 1, \\ k + x & \text{if } x \geq 1. \end{cases}$$
   (a) Compute $\lim\limits_{x \to 1^-} f(x)$ in terms of k.
   (b) Compute $\lim\limits_{x \to 1^+} f(x)$ in terms of k.
   (c) Find the value of k that makes $f(x)$ continuous at $x = 1$.

4. T/F (with justification) The function $f(x) = \begin{cases} \sin x & \text{if } x \leq 0 \\ 1 + \cos x & \text{if } x > 0 \end{cases}$ has a jump discountinuity at $x = 0$.

5. T/F (with justification) A function that is not defined at a point can not be continuous at that point.

6. T/F (with justification) A function that is not continuous at a point can not be defined at that point.