
Calculating Limits Using the Limit Laws

Solutions should show all of your work, not just a single final answer.

1. State the limit laws and the direct substitution property.

2. Let

$$f(x) = \begin{cases} x^2 + 1 & \text{if } x < 1, \\ 4 & \text{if } x = 1, \\ x + 2 & \text{if } 1 < x \leq 2, \\ 6 - x & \text{if } x > 2. \end{cases}$$

(a) Evaluate each of the following, if it exists. (If an answer does not exist, write DNE.)

(i) $\lim_{x \rightarrow 1^-} f(x)$

(ii) $\lim_{x \rightarrow 1^+} f(x)$

(iii) $\lim_{x \rightarrow 1} f(x)$

(iv) $\lim_{x \rightarrow 2^-} f(x)$

(v) $\lim_{x \rightarrow 2^+} f(x)$

(vi) $\lim_{x \rightarrow 2} f(x)$

(b) Sketch the graph of f .

3. Evaluate the limit using algebra and limit laws or say that it does not exist.

(a) $\lim_{x \rightarrow 2} \frac{x^3 - 2}{2x^2 - 3x + 2}$

(b) $\lim_{x \rightarrow -2} \sqrt{x^4 + 3x + 6}$

(c) $\lim_{x \rightarrow 9} \frac{x - 9}{\sqrt{x} - 3}$

(d) $\lim_{x \rightarrow 1} \frac{x^2 + 4x}{x^2 + 3x - 4}$

(e) $\lim_{x \rightarrow 3} \frac{\sqrt{x^2 + 40} - 7}{x - 3}$

(f) $\lim_{x \rightarrow 0} \frac{e^{2x} - 1}{e^x - 1}$

4. Evaluate the limit using algebra and limit laws or say that it does not exist. Note that a represents a constant, and answers can be in terms of a .

(a) $\lim_{t \rightarrow 0} \frac{\sqrt{a+t} - \sqrt{a-t}}{t}$ for $a > 0$

(b) $\lim_{h \rightarrow 0} \frac{1/(a+h)^2 - 1/a^2}{h}$ for $a \neq 0$

5. T/F (with justification) If $\lim_{x \rightarrow 2} g(x) = 0$ and $\lim_{x \rightarrow 2} h(x) = 0$ then $\lim_{x \rightarrow 2} \frac{g(x)}{h(x)}$ does not exist.