# Antiderivatives and Definite Integrals

Name: _____ Section No: _____

1. Find the most general antiderivative of the function (use $C$ as any constant).

   (a) $f(x) = \dfrac{1}{2} + \dfrac{3}{4}x^2 - \dfrac{4}{5}x^3$

   (b) $f(t) = \dfrac{t^4 + 3\sqrt{t}}{t^2}$

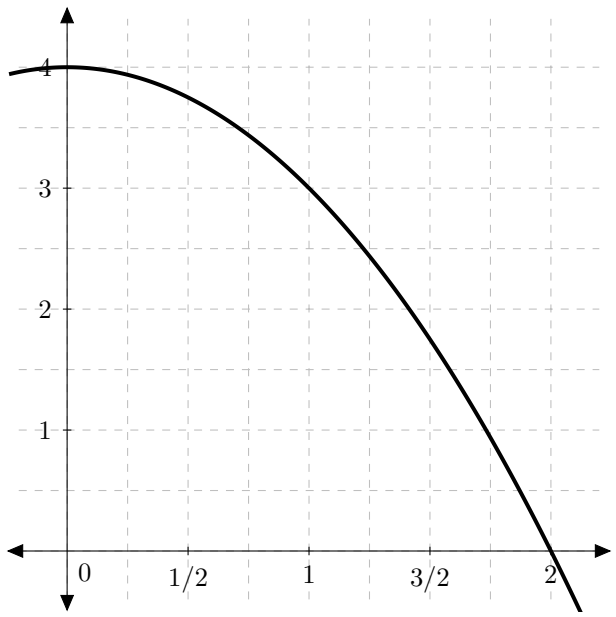
   (c) $g(\theta) = \cos\theta - 5\sin\theta$

2. Find a function $f(x)$ satisfying the given conditions.

   (a) $f'''(x) = \cos x$, $f(0) = 1$, $f'(0) = 2$, and $f''(0) = 3$

   (b) $f''(x) = 2 - 12x$, $f(0) = 9$, $f(2) = 7$

3. The graph of $y = 4 - x^2$ over the interval $[0, 2]$ is given below.



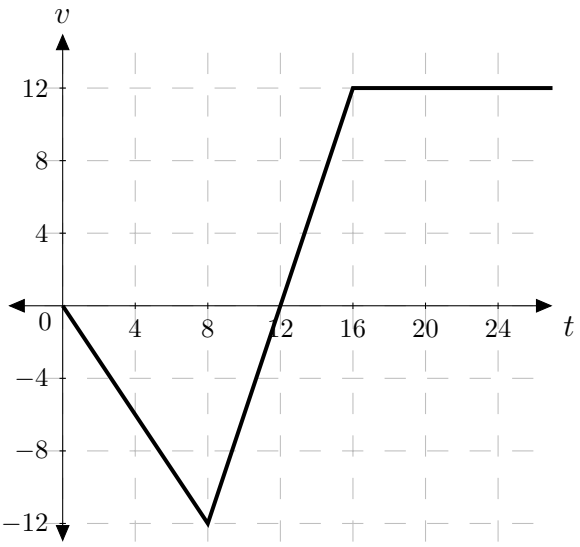

   (a) Estimate the area under the graph over $[0, 2]$ using 4 rectangles and right endpoints: sketch the rectangles and then compute their areas.

(b) Repeat part (a) using 4 rectangles and left endpoints.

(c) Repeat part (a) using 4 rectangles and midpoints.

(d) Compute the exact area under $y = 4 - x^2$ over $[0, 2]$ using the Fundamental Theorem of Calculus and indicate which of the approximations in (a), (b), and (c) is closest to this.

4. Here's a graph of the velocity (in ft/sec) of an object that is moving along a horizontal line with the right direction being positive and the left direction being negative.




Describe the motion of the object over the interval $0 \le t \le 24$: when is it moving left or right, and when is it speeding up or slowing down?

5. (a) Let $A_0(x) = \int_0^x (1 - t^2)\,dt$, $A_1(x) = \int_1^x (1 - t^2)\,dt$, and $A_2(x) = \int_2^x (1 - t^2)\,dt$. Compute these explicitly in terms of $x$ using Part 2 of the Fundamental Theorem of Calculus.

(b) Over the interval $[0, 2]$, use your answers in part (a) to sketch the graphs of $y = A_0(x)$, $y = A_1(x)$, and $y = A_2(x)$ on the same set of axes.

(c) How are the three graphs in part (a) related to each other? In particular, what does Part 1 of the Fundamental Theorem of Calculus tell you about the graphs in part (a)?

(d) On a graph of $y = 1 - t^2$, for $0 \le t \le 2$, shade the region with signed area $A_0(1.5)$. Indicate with + and − which area counts positively and which negatively.

6. Use a definite integral to write down a function $g(x)$ such that $g'(x) = \cos^3 x$ and $g(0) = 1$. Explain why your answer fits the required conditions.