Applications of Derivatives

Name: _____ Section No: _____

Logarithmic Differentiation

1. Use logarithmic differentiation to compute dy/dx. Write final answers in terms of x.

   (a) $y = x^{\cos x}$ (b) $y = x^{2x}$ (c) $y = (x+1)^x$.

Exponential Growth and Decay

2. The element Unobtainium has a half-life of 3 years.

   (a) Determine the differential equation governing the decay of an initial mass of Unobtainium, measuring time in years. Your equation should not have any undetermined parameters in it.

   (b) In how many years will a 14 kg mound of Unobtainium shrink to 1 kg? Round your answer to the nearest tenth.

3. Starbucks serves coffee at 180^o and room temperature in Starbucks is 75^o.

   (a) If the coffee at time $t = 0$ (in minutes) has temperature 180^o, write down a differential equation for its temperature $T(t)$ at time $t > 0$. Your equation will involve an unknown "k" that depends on the coffee.

   (b) If the coffee is 120^o after 10 minutes, how much additional time (from this moment) will it take for the temperature of the coffee to reach 100^o? Round your answer to the nearest tenth.

Newton's Method

4. On the graph of $y = x^2 - 3$ (see Figure 1), draw two successive tangent lines in Newton's method for solving $x^2 - 3 = 0$ when $x_1 = 1$.

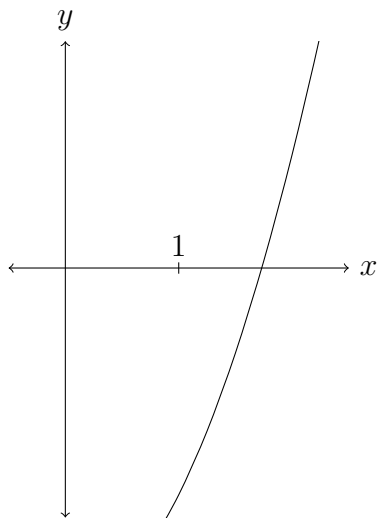

Figure 1: Graph of $y = x^2 - 3$.

5. The solutions to $\sin x = 0$ are $x = \pi k$, where k is an integer (see Figure 2). In particular, the smallest positive solution to $\sin x = 0$ is $x = \pi$.

   You will use Newton's method for $\sin x = 0$ to make numerical estimates for π.

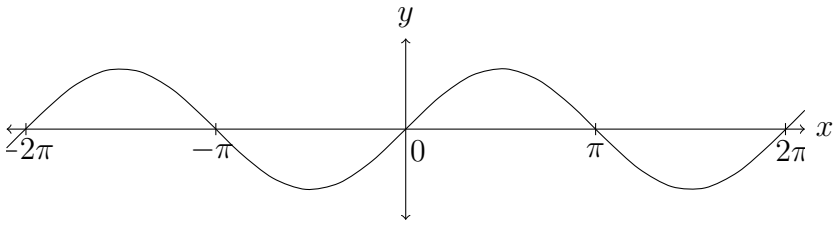

Figure 2: Graph of $y = \sin x$.

(a) What is the recursion formula for Newton's method to solve $\sin x = 0$?

(b) Using Newton's method for $\sin x = 0$ with $x_1 = 3$, tabulate x_n to find the first n for which x_n and x_{n+1} agree to 5 digits after the decimal point. (Be sure you compute trigonometric functions using radians, *not* degrees!)

(c) For the n you found in part b, to how many digits after the decimal point does x_n actually agree with π?

6. In Figure 3 is the graph of $f(x) = \ln(x) - 1$ for $0 < x < 4$. It crosses the x-axis at $x = e$.

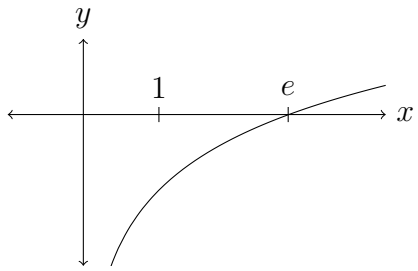

Figure 3: Graph of $y = \ln(x) - 1$.

(a) Write out the recursion for Newton's method used to solve $f(x) = 0$.

(b) Using Newton's method for the equation $\ln(x) - 1 = 0$ with $x_1 = 1$, tabulate x_n to find the first n for which x_n and x_{n+1} agree to 5 digits after the decimal point.

(c) For the n you found in part b, to how many digits after the decimal point does x_n actually agree with e?