Math 1131 Worksheet 4 **Due**: 2nd discussion section in the week of Oct. 12

More Derivatives

Name: ______________________________ Section No: __________

In the problems below, the parameters a and b are constants.

Use the **chain rule** to differentiate y with respect to x.

1. $y = (x^3 - x + 1)^{10}$

2. $y = \sqrt{x^3 + 4x}$

3. $y = e^{ax}\cos(bx)$ (factor out common terms in final answer)

Use **implicit differentiation** to differentiate y with respect to x. Your formula for y' may involve both x and y.

4. $x^2y - axy^2 = x + y$

5. $e^{xy} = x^2 + y^2$

6. $\sin(x + y) = x + \cos(3y)$

Differentiate the following functions **with respect to** t, where r is a function of t.

7. $(r^2 + 1)^4$

8. $\sin(2r) - 2\sin r$

9. $\sqrt{ar + b}$.

10. e^{r^2+ar+b}

Related Rates (In each problem draw a picture, find equations that connect the variables, and differentiate with respect to time.)

11. The radius r of a spherical balloon is expanding at the rate of 14 in/min.

    (a) Determine the rate at which the volume V changes with respect to time, in in^3/min, at the instant when $r = 8$ inches. Round your answer to the nearest integer. Recall that $V = \frac{4}{3}\pi r^3$.

    (b) Determine the rate at which the surface area S changes with respect to time, in in^2/min, when $r = 8$ in. Round your answer to the nearest integer. Recall that $S = 4\pi r^2$.

    (c) If the radius doubles, does dV/dt double? Does dS/dt double?

12. Water is flowing into an upside-down right circular cone with height 3 m and radius 2 m at the top.

    (a) When the water fill the cone up to a height of h m. with a radius of r m. at its surface, express h in terms of r and then the volume V of water in terms of h alone (no r in the formula). Recall that the volume of a right-circular cone with height h and radius r is $\frac{1}{3}\pi r^2 h$.

    (b) Express dV/dt in terms of h and dh/dt. If dV/dt is constant (and not zero), explain from your formula why dh/dt can't be constant too.

    (c) Assuming the water flows into the cone at a constant rate of 2 m^3/min, how quickly is its height changing, in m/min., when the height is 2 m? Round your answer to the nearest tenth.

13. A cop sits in a parked car 10 feet off of North Eagleville Road. (Idealize the road as a straight line, with no width.) As you drive along North Eagleville, the cop trains a radar gun at your car. Let s be the distance of your car from the cop in feet. The radar gun measures the rate at which your distance from the police car is changing with respect to time, which is ds/dt.
    Let x be the distance, in feet, of your car from the point on North Eagleville that is closest to the cop, so your car's velocity is dx/dt.

    (a) Express dx/dt in terms of ds/dt, s, and x.

    (b) Use part a to explain why $ds/dt < dx/dt$. Thus the radar gun's measurement of ds/dt always *underestimates* your car's velocity. (This is why if the radar gun measures a speed greater than the speed limit on the road, the car is going faster and thus deserves a ticket.)

    (c) Does the conclusion in part b depend on the squad car being 10 feet, rather than some other (positive) distance, from the road?