

Limits

Name: _____

Section No: _____

Velocity

The displacement of an object on a line, in meters, is $s = 1 + 2t + \frac{1}{4}t^2$, where t is in seconds.

1. Find the average velocity in m/sec over each of the following time periods. For parts a, b, and c, give the answer to 4 digits after the decimal point. In part d, h is a nonzero variable.

(a) $[1, 1.5]$

(b) $[1, 1.1]$

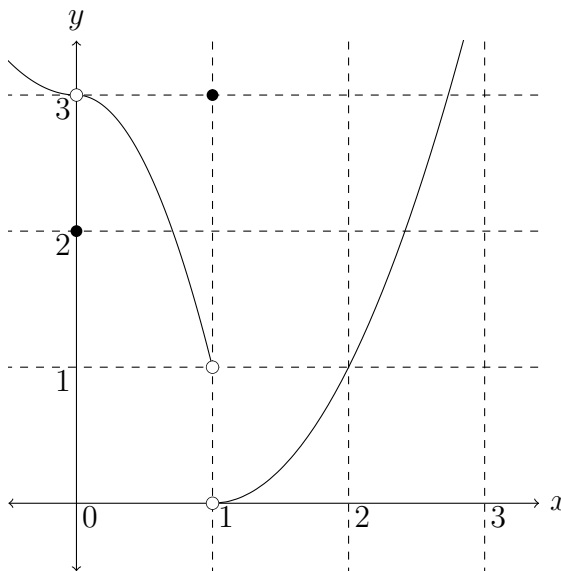
(c) $[1, 1.01]$

(d) $[1, 1 + h]$

2. Find the instantaneous velocity at time $t = 1$, in m/sec.

The Limit of a Function

3. The graph of $y = f(x)$ is below. Compute each value or explain why it doesn't exist.



(a) $\lim_{x \rightarrow 0^-} f(x)$

(b) $\lim_{x \rightarrow 0^+} f(x)$

(c) $\lim_{x \rightarrow 0} f(x)$

(d) $f(0)$

(e) $\lim_{x \rightarrow 1^-} f(x)$

(f) $\lim_{x \rightarrow 1^+} f(x)$

(g) $\lim_{x \rightarrow 1} f(x)$

(h) $f(1)$

(i) $\lim_{x \rightarrow 2^-} f(x)$

(j) $\lim_{x \rightarrow 2^+} f(x)$

(k) $\lim_{x \rightarrow 2} f(x)$

(l) $f(2)$

4. Determine whether the following limits are finite, ∞ , or $-\infty$. Discuss them graphically, numerically, and algebraically.

$$(a) \lim_{x \rightarrow 1} \frac{x+2}{10(x-1)}$$

$$(b) \lim_{x \rightarrow 1} \frac{\sqrt{x}}{2(x-1)^2}$$

Using Limit Laws

5. Evaluate the limit if it exists using algebra and limits laws. Then check the result numerically.

$$(a) \lim_{x \rightarrow 1} \frac{x^2 + 4x}{x^2 + 3x - 4}$$

$$(b) \lim_{x \rightarrow 3} \frac{\sqrt{x^2 + 40} - 7}{x - 3}$$

6. Evaluate the limit in terms of a , if the limit exists, using algebra and limit laws.

$$(a) \lim_{t \rightarrow 0} \frac{\sqrt{a+t} - \sqrt{a-t}}{t} \text{ for } a > 0$$

$$(b) \lim_{h \rightarrow 0} \frac{1/(a+h)^2 - 1/a^2}{h} \text{ for } a \neq 0$$

Continuity

7. Determine the exact value of the constant a making the following function continuous at 1:

$$f(x) = \begin{cases} \frac{x^2 + 2x - 3}{x^2 + 5x - 6} & \text{if } x \neq 1, \\ a & \text{if } x = 1. \end{cases}$$

8. Let

$$g(x) = \begin{cases} x^2 + x & \text{if } x < 1, \\ a & \text{if } x = 1, \\ x - 1 & \text{if } x > 1. \end{cases}$$

- (a) Determine the value of a for which g is continuous from the left at 1.
(b) Determine the value of a for which g is continuous from the right at 1.
(c) Is there a value of a for which g is continuous at 1? Explain.

Limits at Infinity and Horizontal Asymptotes

Find the limit in each case or explain why it does not exist (and if it is $\pm\infty$).

$$9. \lim_{x \rightarrow \infty} \frac{x^2}{\sqrt{6x^4 - 1}}$$

$$10. \lim_{x \rightarrow \infty} x + \sqrt{x^2 + 2x}$$

$$11. \lim_{x \rightarrow \infty} \frac{100000x}{x^3 + x}$$

$$12. \lim_{x \rightarrow \infty} \frac{e^{3x} - e^{-3x}}{e^{3x} + e^{-3x}}$$