Read This First!

- Please read each question carefully. In order to receive full credit on a problem, solution methods must be complete, logical and understandable.

- Answers must be clearly labeled in the spaces provided after each question. Please cross out or fully erase any work that you do not want graded. The point value of each question is indicated after its statement. No books or other references are permitted.

- Give any numerical answers in exact form, not as approximations. For example, one-third is \( \frac{1}{3} \), not .33 or .33333. And one-half of \( \pi \) is \( \frac{1}{2} \pi \), not 1.57 or 1.57079.

- Turn smart phones, cell phones, and other electronic devices off (not just in sleep mode) and store them away.

- Calculators are allowed but you must show all your work in order to receive credit on the problem.

- If you finish early then you can hand in your exam early.

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1. If the statement is always true, circle the printed capital T. If the statement is sometimes false, circle the printed capital F. In each case, write a careful and clear justification or a counterexample.

(a) \( \ln(2^{3x}) = 3 \ln(2^x) \).

T F [3]

Justification:

(b) \( \frac{d}{dx}(\sin^3 x) = 3 \sin^2 x \cos x \)

T F [3]

Justification:

(c) If \( f'(x) = \ln x \) then \( (f(x^2))' = 4x \ln x \).

T F [3]

Justification:

(d) \( \int_2^x \frac{2x}{x^2 + 1} \, dx = \ln(x^2 + 1) + C. \)

T F [3]

Justification:

(e) If \( \frac{d}{dx} \int_2^5 x^2 \, dx = x^2 \).

T F [3]

Justification:
2. Use calculus to compute the following limits. If the limit does not exist, write DNE.

(a) \( \lim_{x \to 2} \frac{4^x - x^4}{x^2 - 4} \)

(b) \( \lim_{x \to \infty} \frac{1000 \ln x}{x^3} \)
3. Compute antiderivatives of the following functions using antidifferentiation rules.

(a) \( f(x) = e^{kx} \frac{1}{x^2} \), where \( k \) is a nonzero constant.

(b) \( f(x) = x \sqrt{x^2 - 5} \)

(c) \( f(x) = (\sin x)(\cos^2 x) \)
4. Integration.

(a) Compute $\int_{0}^{2} e^{3x} \, dx$

(b) Compute $\int_{1}^{b} (x^3 - x) \, dx$, where $b$ is a constant.

(c) Express $\int_{1}^{3} x\sqrt{x^2 - 1} \, dx$ as a definite integral in terms of $u = x^2 - 1$, but do not evaluate the integral.
5. Use calculus to find the equation of the tangent line to the curve $xy + y^3 = 14$ at the point $(3, 2)$. 
6. A helicopter rises vertically so that at time $t$ its height is $h(t) = t^2 + t$, where $t$ is measured in seconds and $h(t)$ is measured in meters. At the time when the height of the helicopter is 20, what is its velocity (in m/sec) and acceleration (in m/sec$^2$)? (Hint: First you need to find the time when the height is 20.)
7. Let $f(x) = x^4 - 8x^2$. Use calculus to find the open intervals on which $f$ is increasing or decreasing, the local maximum and minimum values of $f$, the intervals of concavity and the inflection points.
8. A dead body is found in a room where the thermostat is $65^\circ$. The body’s temperature is measured to be $85^\circ$, and an hour later it is $83^\circ$. Use Newton’s Law of Cooling to estimate how long ago, in minutes from when it was first found, the body died. Assume the temperature of the person, when last alive, was $98.6^\circ$. 


9. After cutting out square corners from a square piece of cardboard, as in the figure below, what remains can be folded up to form a rectangular box with an open top. Find the dimensions of the original rectangular cardboard and the side length of the cut corners that produces a box with volume 36 in$^3$ and requiring the least amount of initial cardboard.
10. In the figure below, draw rectangles in the Riemann sum approximation to $\int_1^3 f(x) \, dx$ using 4 rectangles with right endpoints. Label numerically where the sides of the rectangles meet the $x$-axis.