Section A.2
Section Objectives:

- Know what polynomials and rational functions are.
- Add, subtract and multiply polynomials.
- Factor polynomials (quadratic, difference of squares)
- Add, subtract, multiply and divide rational expressions.
- Solve equations involving polynomials and rational expressions.
- Simplify rational expressions using rationalization.

Practice Problems

1. Let

$$f(x) = 2x^2 + 3x \qquad g(x) = 2x^2 - 7x + 6 \qquad h(x) = 4x + 1$$

(a) Find and simplify $f(x) - g(x)$.

$2x^2 + 3x - (2x^2 - 7x + 6) = 2x^2 + 3x - 2x^2 + 7x - 6$

$= \boxed{10x - 6}$

(b) Find and simplify $f(x) \cdot h(x)$.

$(2x^2 + 3x)(4x + 1) = 8x^3 + 12x^2 + 2x^2 + 3x$

$= \boxed{8x^3 + 14x^2 + 3x}$

(c) Find and simplify $g(x) \cdot h(x)$.

$(2x^2 - 7x + 6)(4x + 1) = 8x^3 - 28x^2 + 24x + 2x^2 - 7x + 6$

$= \boxed{8x^3 - 26x^2 + 17x + 6}$

(d) Factor $f(x)$ completely.

$f(x) = 2x^2 + 3x = \boxed{x(2x + 3)}$

(e) Factor $g(x)$ completely.

$g(x) = 2x^2 - 7x + 6 = \boxed{(x - 2)(2x - 3)}$

(f) Find and simplify $\frac{f(x)}{h(x)} + \frac{h(x)}{f(x)}$.

$$\frac{f(x)}{h(x)} + \frac{h(x)}{f(x)} = \frac{(2x^2+3x)}{(4x+1)} + \frac{(4x+1)}{(2x^2+3x)} = \frac{(2x^2+3x)}{(2x^2+3x)} \cdot \frac{(2x^2+3x)}{(4x+1)} + \frac{(4x+1)}{(2x^2+3x)} \cdot \frac{(4x+1)}{(4x+1)}$$

$$= \frac{4x^4 + 6x^3 + 6x^3 + 9x^2 + 16x^2 + 4x + 4x + 1}{8x^3 + 14x^2 + 3x} \quad \text{(see part b)}$$

$$= \boxed{\frac{4x^4 + 12x^3 + 25x^2 + 8x + 1}{8x^3 + 14x^2 + 3x}}$$

(g) Find and simplify $\frac{\frac{f(x)}{g(x)}}{h(x)}$.

$$\frac{\frac{(2x^2+3x)}{2x^2-7x+6}}{4x+1} = \frac{(2x^2+3x)}{(2x^2-7x+6)} \cdot \frac{1}{(4x+1)} = \boxed{\frac{2x^2+3x}{8x^3-26x^2+17x+6}} \quad \text{(see part c)}$$

(h) Find and simplify $\frac{f(x)}{\frac{g(x)}{h(x)}}$.

$$= \frac{2x^2+3x}{\frac{2x^2-7x+6}{4x+1}} = \frac{2x^2+3x}{1} \cdot \frac{4x+1}{2x^2-7x+6} = \boxed{\frac{8x^3+14x^2+3x}{2x^2-7x+6}} \quad \text{(see part b)}$$

2. Simplfy $x^2 - 4y^2$.

$x^2 - 4y^2 = (x - 2y)(x + 2y)$

(difference of two squares)

3. Solve for s: $s^4 - 9 = 0$.

$s^4 - 9 = 0$

$(s^2 - 3)(s^2 + 3) = 0$

so $s^2 - 3 = 0 \Rightarrow s^2 = 9$ $\quad s = \pm\sqrt{3}$

or $s^2 + 3 = 0 \Rightarrow s^2 = -9$ impossible.

or $s^4 = 9$

$s = \pm\sqrt[4]{9}$

$s = \pm\sqrt{3}$

4. Simplify the following expression by rationalizing the numerator. Let $f(x) = \sqrt{x}$

$$\frac{f(x) - f(4)}{x - 4}.$$

$$\frac{f(x) - f(4)}{x - 4} = \frac{\sqrt{x} - 2}{x - 4} = \frac{(\sqrt{x} - 2)(\sqrt{x} + 2)}{(x - 4)(\sqrt{x} + 2)} = \frac{(x - 4)}{(x - 4)(\sqrt{x} + 2)}$$

$$= \frac{1}{\sqrt{x} + 2}$$

More Practice from Textbook A.2: You should do as many problems from each set (1-14, 15-26,27-38, 39-46, 47-58, 59-66 and 67-70), as needed until you are comfortable with these techniques.