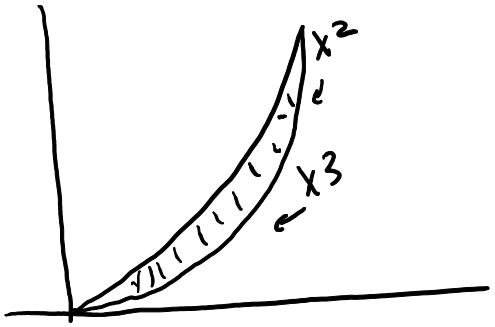
Section 6.6: Area Between Two Curves
Section Objectives:

- Use definite integrals to find the area between two curves by integrating the top minus the bottom.
- Find the area of a curve under the x-axis.
- Be careful with determining which is the top curve an which is the bottom.

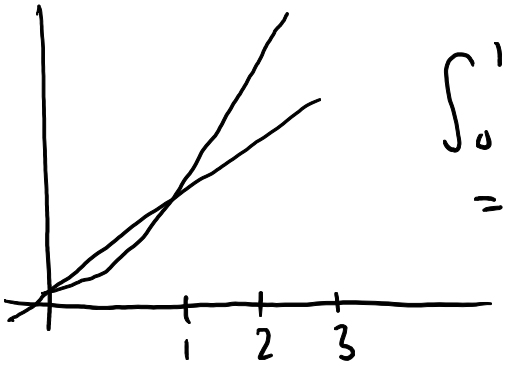
Practice Problems

1. Find the area between $f(x) = x^2$ and $g(x) = x^3$ from $x = 0$ to $x = 1$.



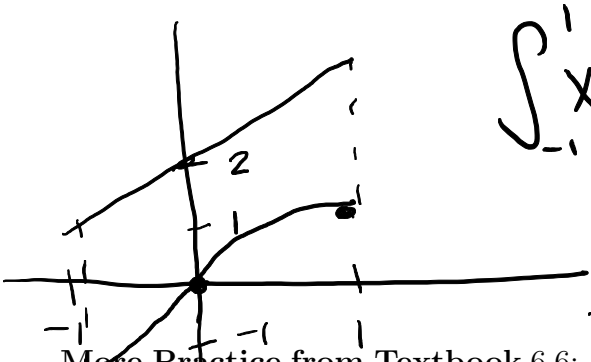
$$\int_0^1 x^2 - x^3 \, dx = \frac{x^3}{3} - \frac{x^4}{4}\Big|_0^1 = \frac{1}{3} - \frac{1}{4} = \boxed{\frac{1}{12}}$$

2. Find the area between $f(x) = x$ and $g(x) = x^2$ from $x = 0$ to $x = 3$. Be careful with determining which curve is the bottom and which is the top.



$$\int_0^1 x - x^2 \, dx + \int_1^3 x^2 - x \, dx$$
$$= \frac{x^2}{2} - \frac{x^3}{3}\Big|_0^1 + \frac{x^3}{3} - \frac{x^2}{2}\Big|_1^3$$
$$= \left(\frac{1}{2} - \frac{1}{3}\right) + \left[9 - \frac{9}{2} - \left(\frac{1}{3} - \frac{1}{2}\right)\right]$$
$$= \frac{1}{6} + \frac{9}{2} + \frac{1}{6} = \boxed{\frac{29}{6}}$$

3. Find the area between $f(x) = x + 2$ and $g(x) = \sqrt[3]{x}$ from $x = -1$ to $x = 1$.



$$\int_{-1}^1 x + 2 - x^{1/3} \, dx$$
$$\frac{x^2}{2} + 2x - \frac{3x^{4/3}}{4}\Big|_{-1}^1 = \boxed{4}$$
$$= (1/2 + 2 - 3/4) - (1/2 - 2 - 3/4)$$

More Practice from Textbook 6.6: You should do as many problems from each set (1-36, 41, 42, 50-53), as needed until you are comfortable with these techniques. 50-53 are good practice for application problems. (We are skipping Lorentz Curves.)