

Section 6.5: The Fundamental Theorem of Calculus

Section Objectives:

- Know the following Properties of Definite Integrals:

$$- \int_a^a f(x) dx = 0$$

$$- \int_a^b k f(x) dx = k \int_a^b f(x) dx \text{ where } k \text{ is a constant}$$

$$- \int_a^b f(x) \pm g(x) dx = \int_a^b f(x) dx \pm \int_a^b g(x) dx$$

$$- \int_a^b f(x) dx = \int_a^c f(x) dx + \int_c^b f(x) dx \text{ where } a < c < b.$$

$$- \int_b^a f(x) dx = - \int_a^b f(x) dx$$

- Know the statement of the Fundamental Theorem of Calculus, Part 1, i.e if $f(x)$ is continuous on $[a, b]$:

$$\frac{d}{dx} \int_a^x f(t) dt = f(x)$$

- Know the statement of the Fundamental Theorem of Calculus, Part 2, i.e. if $f(x)$ is continuous on $[a, b]$:

$$\int_a^b f(x) dx = F(b) - F(a)$$

where $F(x)$ is any antiderivative of $f(x)$.

- Use FTC 2 to evaluate definite integrals.
- Use definite integrals to find the overall change in a function from its rate of change (i.e. change in cost from marginal cost, number sold from rate of sale, etc.).

Practice Problems

1. Assume $\int_0^3 f(x) dx = 5$, $\int_3^4 f(x) dx = 2$ and $\int_0^3 g(x) dx = 4$. Evaluate the following:

(a) $\int_0^3 4f(x) dx$

(b) $\int_0^3 f(x) - 2g(x) dx$

(c) $\int_0^4 f(x) dx$

(d) $\int_3^0 4g(x) dx$

2. Let $f(x) = \int_3^x e^{3t} dt$. Find $f(3)$ and $f'(x)$.

3. Evaluate the following definite integral using FTC 2.

(a) $\int_0^2 x^2 dx$

(b) $\int_1^4 \frac{3}{x^2} dx$

(c) $\int_2^5 \frac{e^x}{1+e^x} dx$

(d) $\int_2^7 \frac{2x}{3x^2+1} dx$

4. Mustafa finds that the rate of sales of an item is given by $S'(t) = -3t^2 + 36t$ where t is number of weeks after an advertising campaign. How many items are sold during the third week?

More Practice from Textbook 6.5: You should do as many problems from each set (1-24, 35-38, 39-40, 41-57), as needed until you are comfortable with these techniques. 41-57 are good practice for application problems. (We are skipping average value of a function.)