Section 6.4: The Definite Integral
Section Objectives:

- Know how to find the left-hand and right-hand sums to estimate the area under a curve (by hand, with a small number of rectangles). You will not be asked to estimate using a large number of rectangles on a quiz/ exam.
- Know the definition of the definite integral (defined as area under the curve, found as a limit of right-hand or left-hand sums as the number of rectangles goes to ∞.)
- Find the exact area of definite integrals using geometry.
- Know that the distance traveled by an object is the area under it velocity curve.
- Know the three order properties of integrals:
  - If $f(x) \geq 0$, then $\int_a^b f(x)\,dx \geq 0$
  - If $f(x) \leq g(x)$ for $a \leq x \leq b$, then $\int_a^b f(x)\,dx \leq \int_a^b g(x)\,dx$.
  - If $m \leq f(x) \leq M$ for $a \leq x \leq b$, then

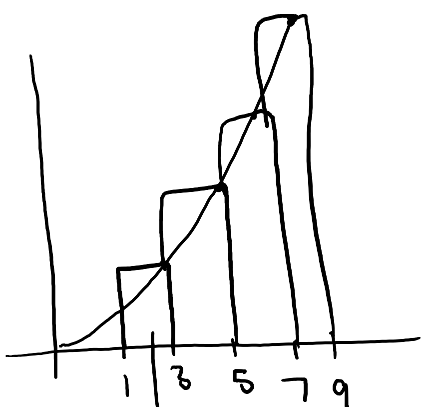
$$m(b-a) \leq \int_a^b f(x)\,dx \leq M(b-a).$$

Practice Problems

1. Estimate $\displaystyle\int_1^9 x^2 + 2x\,dx$ using $n = 4$ rectangles and right endpoints. Then use left endpoints with $n = 4$.

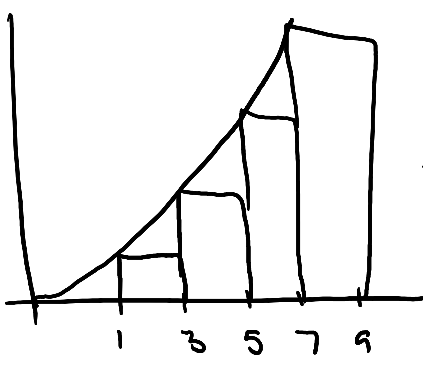
width of rectangle $\frac{9-1}{4} = \frac{8}{4} = 2$

Right endpoints



$2f(3)+2f(5)+2f(7)+2f(9) = 2(3^2+6+5^2+10+7^2+14+9^2+18)$
$= 424$

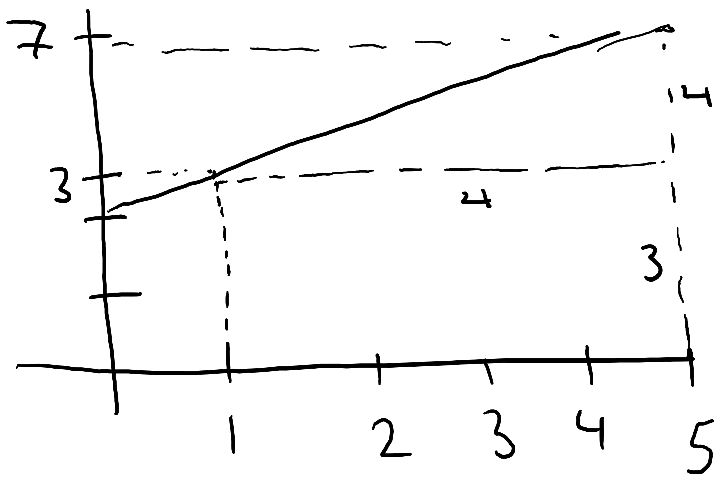
left endpoints



$= 2f(1)+2f(3)+2f(5)+2f(7)$
$= 2(1^2+2+3^2+6+5^2+10+7^2+14)$
$= 232$

2. Find the value of the definite integral by finding the area of the appropriate geometric shape.

$$\int_1^5 x + 2\, dx$$



triangle

$\frac{1}{2}(4)(4) = 8$

rectangle

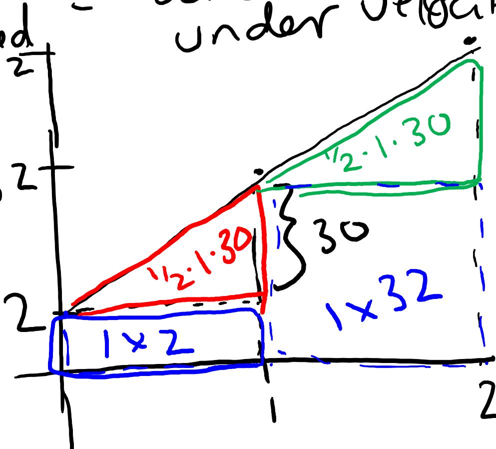
$4 \times 3 = 12$

total area $\quad 20$

$$\int_1^5 x + 2\, dx = 20$$

3. Munir is driving his car with a velocity (in miles per hour) of $v(t) = 30t + 2$ for $0 \le t \le 2$. How far does he drive during the first hour? During the second hour?

Distance traveled = area under velocity curve



1st hour: $\int_0^1 v(t)\,dt$ = area rectangle + area triangle = $2 + 15 = 17$ mi

2nd hour: $\int_1^2 v(t)\,dt = 1 \times 32 + \frac{1}{2} \cdot 1 \cdot 30 = 32 + 15 = 47$ mi

4. We want to try to get a lower and upper bound on $\int_{-1}^{1} e^x(1-x)$.

(a) First, we want to find lower and upper bounds on the function $f(x) = e^x(1-x)$. To do this, we find the absolute minimum and maximum value of $f(x)$ on $[-1, 1]$. Find these.

min/max occur at CPs or endpoints.

$$f'(x) = e^x(-1) + e^x(1-x)$$
$$= e^x(\ x\ ) = 0$$
$$x = 0$$

Test

x	$f(x)$
-1	$2/e = .736$
0	$1 \leftarrow$ max
1	$0 \leftarrow$ min

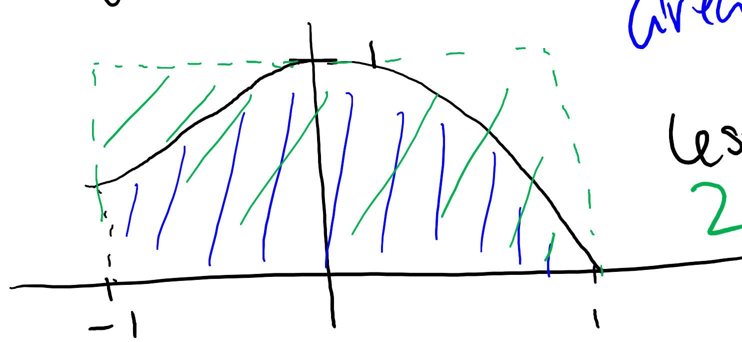
(b) Now, use these to find bounds on the integral of the function. Illustrate with a picture.

We know $0 \le e^x(1-x) \le 1$

so $0(1+1) \le \int_{-1}^{1} e^x(1-x) \le 1(1+1)$

$$0 \le \int_{-1}^{1} e^x(1-x) \le 2$$

Picture:



Area we want (blue) less than 2 (green area).

More Practice from Textbook 6.4: You should do as many problems from each set (1-22 (will only be asked to do with small number of rectangles), 23-32, 33-42, 43-50,53-54), as needed until you are comfortable with these techniques.