Section 6.4: The Definite Integral
Section Objectives:

- Know how to find the left-hand and right-hand sums to estimate the area under a curve (by hand, with a small number of rectangles). You will not be asked to estimate using a large number of rectangles on a quiz/ exam.
- Know the definition of the definite integral (defined as area under the curve, found as a limit of right-hand or left-hand sums as the number of rectangles goes to ∞.)
- Find the exact area of definite integrals using geometry.
- Know that the distance traveled by an object is the area under it velocity curve.
- Know the three order properties of integrals:
  - If $f(x) \geq 0$, then $\int_a^b f(x)\,dx \geq 0$
  - If $f(x) \leq g(x)$ for $a \leq x \leq b$, then $\int_a^b f(x)\,dx \leq \int_a^b g(x)\,dx$.
  - If $m \leq f(x) \leq M$ for $a \leq x \leq b$, then

$$m(b-a) \leq \int_a^b f(x)\,dx \leq M(b-a).$$

Practice Problems

1. Estimate $\int_1^9 x^2 + 2x\,dx$ using $n = 4$ rectangles and right endpoints. Then use left endpoints with $n = 4$.

2. Find the value of the definite integral by finding the area of the appropriate geometric shape.

$$\int_1^5 x + 2\,dx$$

3. Munir is driving his car with a velocity (in miles per hour) of $v(t) = 30t + 2$ for $0 \leq t \leq 2$. How far does he drive during the first hour? During the second hour?

4. We want to try to get a lower and upper bound on $\int_{-1}^{1} e^x(1-x)$.

   (a) First, we want to find lower and upper bounds on the function $f(x) = e^x(1-x)$. To do this, we find the absolute minimum and maximum value of $f(x)$ on $[-1, 1]$. Find these.

   (b) Now, use these to find bounds on the integral of the function. Illustrate with a picture.

More Practice from Textbook 6.4: You should do as many problems from each set (1-22 (will only be asked to do with small number of rectangles), 23-32, 33-42, 43-50,53-54), as needed until you are comfortable with these techniques.