Section 6.2: Substitution
Section Objectives:

- Use substitution to find antiderivatives.
- Identify the appropriate u for substitution.
- Know the connection between substitution and the chain rule.

Practice Problems

1. Evaluate the following antiderivatives:

(a) $\int 4(2x+1)^6\,dx$

$u = 2x+1 \quad du = 2\,dx \quad dx = \frac{du}{2}$

$$= \int 4(u)^6 \cdot \frac{du}{2} = 2\int u^6\,du = 2\frac{u^7}{7} + C = \boxed{\frac{2(2x+1)^7}{7} + C}$$

Check by taking derivative: $\frac{d}{dx}\left(\frac{2(2x+1)^7}{7} + C\right) = \frac{2\cdot 7(2x+1)^6}{7}\cdot 2 = 4(2x+1)^6$ ✓

(b) $\int x(3-x^2)^4\,dx$

$u = 3 - x^2 \quad du = -2x\,dx \quad dx = \frac{du}{-2x}$

$$\int \cancel{x}(u)^4 \frac{du}{-2\cancel{x}} = -\frac{1}{2}\int u^4\,du = -\frac{1}{2}\cdot\frac{u^5}{5} + C = -\frac{1}{10}(3-x^2)^5 + C$$

Derivative Check: $\frac{d}{dx}\left(-\frac{1}{10}(3-x^2)^5 + C\right) = -\frac{5}{10}(3-x^2)^4(-2x) = x(3-x^2)^4$ ✓

(c) $\int (2x+1)\sqrt[4]{x^2+x}\,dx$

$u = x^2 + x \quad du = (2x+1)\,dx$

$$= \int u^{1/4}\,du = \frac{u^{5/4}}{5/4} + C = \boxed{\frac{4}{5}(x^2+x)^{5/4} + C}$$

Derivative Check: $\frac{d}{dx}\left(\frac{4}{5}(x^2+x)^{5/4} + C\right) = (x^2+x)^{1/4}(2x+1)$ ✓

(d) $\int \frac{e^x}{e^x+5}\,dx$

$u = e^x + 5 \qquad du = e^x\,dx$

$$\int \frac{1}{u}\,du = \ln|u| + C$$

$$= \boxed{\ln|e^x+5| + C}$$

derivative check

$\frac{d}{dx}\left(\ln|e^x+5| + C\right) = \frac{1}{e^x+5}\cdot e^x$ ✓

(e) $\int \frac{1}{x\ln(x)}\,dx$

$u = \ln x \qquad du = \boxed{\frac{1}{x}\,dx}$

$$\int \frac{1}{u}\,du = \ln|u| + C$$

$$= \ln|\ln(x)| + C.$$

Derivative check:

$\frac{d}{dx}\left(\ln(\ln(x))\right) = \frac{1}{\ln(x)}\cdot\frac{1}{x} = \frac{1}{x\ln x}$

2. The Three Michael's Metal Company decided to get into the water bottle making business. Find the cost function if the marginal cost, in dollars , is given by $x^3(200+x^4)$ where x is number of cases produced and the fixed costs are \$300.

$C'(x) = x^3(200+x^4)$

$C(x) = \int x^3(200+x^4)\,dx$

$u = x^4 + 200$

$du = 4x^3\,dx$

$\frac{du}{4} = x^3\,dx$

$$\int u\,\frac{du}{4} = \frac{1}{4}\cdot\frac{u^2}{2} + C = \frac{1}{8}(x^4+200) + C$$

$C(0) = 300 \Rightarrow C(0) = 25 + C \Rightarrow C = 275$

More Practice from Textbook 6.2: You should do as many problems from each set (1-34, 35-48), as needed until you are comfortable with these techniques. 35-48 are good practice for application problems.

$$\boxed{C(x) = \frac{1}{8}(x^4+200) + 275}$$